Introducing Névir-Blender-Nambu Dynamics to the World of Atmospheric Numerical Modelling

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Overview

Using energy as only constituent conserved quantity, canonical Hamiltonian dynamics is not capable to treat enstrophy conservation on the same level. Nambu mechanics as a generalization of Hamiltonian dynamics gives the possibility to represent the evolution of a system using various streamfunctions, which has been applied for atmospheric systems by Névir (1993, 1998).

Possibilities for application in meteorological modelling lie in the grid-point model branch in the style of Arakawa (1966) and Sadourny (1975) as well as in the finite element model branch with the base functions to represent localized vortex structures.

Salmon (2005) suggested a general discretization to the Nambu bracket in which its antisymmetry is preserved and thus conservation quantities are also automatically preserved in the numerical scheme. That conservation property is of great importance in any numerical modelling, and especially in climate simulations.

A thorough application of Salmons ideas to atmospheric modelling generates a new class of numerical schemes. With such a project, Névir's theoretical work (1993, 1998) is applicable to a wide field of atmospheric simulations.

From Hamiltonian to Nambu Mechanics

Systems with zero divergence in phase space:

- Canonical Hamiltonian Systems: Poisson Tensor, one constituent conserved quantity (energy), Poisson brackets
- Nambu systems: Nambu tensor, N constituent conserved quantities (the Casimirs of noncanonical theory), Nambu brackets

Hamiltonian Formalism: implicitly divergence free	Generalization →	Nambu Formalism: implicitly divergence free
$\frac{dx_i}{dt} = P_{ij}\frac{\partial H}{\partial x_j} =: \{x_i, H\}$	Equation of motion	$\frac{dx_i}{dt} = N_{ijk}\frac{\partial H_1}{\partial x_j}\frac{\partial H_2}{\partial x_k} =: \{x_i, H_1, H_2\}$
$\frac{\partial \rho}{\partial t} = -\{\rho, H\}$	Liouville equation	$\frac{\partial \rho}{\partial t} = -\{\rho, H_1, H_2\}$
$\frac{\partial u(\mathbf{r})}{\partial t} = D(u)\frac{\delta \mathcal{H}[u]}{\delta u}$	Field equation	$\frac{\partial u(\mathbf{r})}{\partial t} = N(u)\frac{\delta \mathcal{H}_1[u]}{\delta u}\frac{\delta \mathcal{H}_2[u]}{\delta u}$

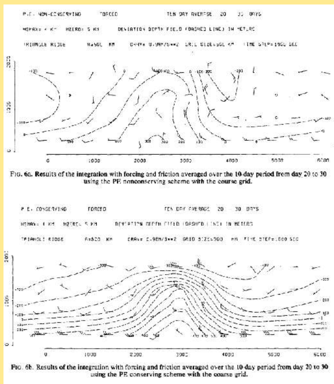
Atmospheric Dynamics in Nambu Representation (Névir)

Atmospheric Model	Nambu Dynamics	Coordinates	Integrals of Motion
2D Incompressible	$\frac{\partial \zeta}{\partial t} = -J(\frac{\delta \mathcal{E}}{\delta \zeta}, \frac{\delta \mathcal{H}}{\delta \zeta})$	Vorticity	Enstrophy, Kinetic Energy
3D Incompressible	$\frac{\partial \xi}{\partial t} = -\nabla \times ((\nabla \times \frac{\delta h}{\delta \xi}) \times (\nabla \times \frac{\delta \mathcal{H}}{\delta \xi}))$	Vorticity Vector	Helicity, Kinetic Energy
Inviscid Adiabatic	$\frac{\partial \mathbf{v}}{\partial t} = \{\mathbf{v}, h_a, \mathcal{H}\} + \{\mathbf{v}, \mathcal{M}, \mathcal{H}\} + \{\mathbf{v}, \mathcal{S}, \mathcal{H}\}$, $\frac{\partial \rho}{\partial t} = \{\rho, \mathcal{M}, \mathcal{H}\}$, $\frac{\partial \sigma}{\partial t} = \{\sigma, \mathcal{S}, \mathcal{H}\}$	Velocity, Density, Entropy Density	Helicity, Mass, Entropy, Total Energy
Quasigeostrophic Shallow Water	$\frac{\partial \Pi}{\partial t} = -J(\frac{\delta \mathcal{E}_p}{\delta \Pi}, \frac{\delta \mathcal{H}}{\delta \Pi})$	Potential Vorticity	Potential Enstrophy, Total Energy
Quasigeostrophic Baroclinic	$\frac{\partial \Pi_{QG}}{\partial t} = -J(\frac{\delta \mathcal{E}_p}{\delta \Pi_{QG}}, \frac{\delta \mathcal{H}}{\delta \Pi_{QG}})$	Potential Quasigeostrophic Vorticity	Potential Enstrophy, Total Energy

Wave-vortex duality

	Mathematical Structure	
Rossby Waves	Nonlinear Solution	Point Vortices
Wave Amplitudes	Dynamics	Vortex Positions
Wave Vectors	Parameters	Circulations
Wave Triad	Base System	Point Vortex Trio
Canonical Nambu	Form	Noncanonical Nambu
Energy, Enstrophy	Conserved Quantities	Energy, Ang. Momentum
$\frac{dX_k}{dt} = \sum_{i,j} \frac{N_{ijk}}{k_i k_j k_k}\frac{\partial H}{\partial X_i}\frac{\partial E}{\partial x_j}$	Equations of motion	$\frac{r_{ij}^2}{dt} = \sum_k \frac{8F_{ijk}\sigma_{ijk}}{\gamma_i\gamma_j\gamma_k}\frac{\partial(E,H)}{\partial(r_{jk}^2, r_{ki}^2)}$
	Numerics	
Wave Vector Space	Discretization	Position Space
	Meteorology	
Global Waves	Phenomena	Localized Vortex Structures
Mean Flow		Blockings, Cut-Off-Vortices
Inverse Energy Cascade	Energy Flow	Point Vortex Clustering

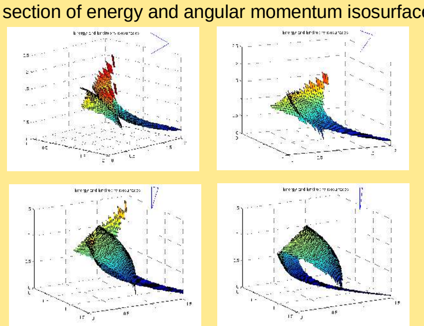
Arakawa's conserving scheme



Simulated flow over a steep ridge without (top) and with (bottom) conservation of potential enstrophy and energy (Arakawa & Lamb, 1981).

Discrete Structures: Point Vortices

- Localized vortex structures: Point vortex dynamics instead of spectral methods (Kuhlbrodt & Névir, 2000)
- Vortex Trio: Dynamics in Nambu form, on intersection of energy and angular momentum isosurface



- Nambu mechanics as geometrical method: Nonlinear dynamics restricted to intersection of isosurfaces

Outline of strategy / MetStroem Project

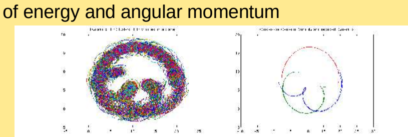
Numerical Application: Gaßmann, Herzog	↔	Theoretical Meteorology Névir, Sommer

Starting with a recapitulation of the history of meteorological modelling that went from the barotropic vorticity equation over baroclinic filtered models and the primitive hydrostatic models up to the nonhydrostatic compressible models, an equivalent hierarchy of Nambu-brackets will be formulated.

Beginning from the most simple form, numerical schemes according to the proposal of Salmon will be implemented into a simulation model and compared with traditional forms of implementations.

Energy flow towards large scales

- Spectral representation: Inverse energy cascade for energy and enstrophy conserving flows
- Many-vortex-system: clustering due to conservation of energy and angular momentum



- Structure formation on large scales: Vortex analogon to inverse energy cascade

References

-A. Arakawa, Computational design for long-term numerical integration of the equations of fluid motion: Two-dimensional incompressible flow. Part I., *J.Comp.Phys.* **1** (1966), 119-143
-A. Arakawa & V. R. Lamb, A Potential Entropshy and Energy Conserving Scheme for the Shallow Water Equations, *M. Weather Rev.* **109** (1981)
-T. Kuhlbrodt und P. Névir, Low-Order Point Vortex Models of Athmospheric Blocking, *Meteorol. Atmos. Phys.* **73** (2000) 127—138.
-Y. Nambu, Generalized Hamiltonian Dynamics, *Phys. Rev. D* **7.3** (1973) 2405-2412
-P. Névir, Die Nambu-Felddarstellung der Hydro-Thermodynamik und ihre Bedeutung für die dynamische Meteorologie, *Habilitatiosschrift FB Geowissenschaften*, Freie Universität Berlin, 1998.
-P. Névir und R. Blender, A Nambu representation of incompressible hydrodynamics using helicity and enstrophy, *J. Phys. A: Math. Gen.* **26** (1993) L1189—L1193.
-R. Sadourny, The dynamics of finite-difference models of the shallow water equations, *J.Atmos.Sci* **32** (1975), 680-689
-R. Salmon, A general method for conserving quantities related to potential vorticity in numerical methods, *Nonlinearity* **18** (2005), R1 – R16