Towards a Power Law Distribution of Tornadoes and Cyclones and the relation to the Gutenberg-Richter Law of Earthquakes

Lisa Schielicke and Peter Névir
Institut für Meteorologie, Freie Universität Berlin

Freie Universität Berlin

INTRODUCTION

Various definitions of the **intensity** of atmospheric vortices exist. These definitions are based on different local parameters, e.g. the intensity of tornadoes is defined by their damage-related **wind speeds** (Fujita-scale, Enhanced-Fujita-scale), cyclones are mostly investigated concerning their minimum **central pressure**, or their maximum wind speed.

Do these local parameters really describe the systems as a whole? What methods are applied in other geosciences to handle this problem?

Peering into seismology, one finds the concept of the seismic moment, which contains all information about the physical parameters of an earthquake source and the rupture process.

The seismic moment is also related to the energy released during the quake (e.g. Stein and Wysession, 2002).

Analogous to this concept a moment of tornadoes and other atmospheric vortices is introduced. **This pictures a consideration of the process, rather than the local state**

SEISMIC MOMENT CONCEPT

Earthquakes are considered as slip on a fault causing the radiation of seismic waves.

The **seismic moment tensor** is a mathematical representation containing forces that yield the same seismic wave pattern. The seismic moment tensor:

- contains nine force couples, is symmetric, six independent components
- represents fault geometry (via the different components) and size of an earthquake (via its magnitude = scalar seismic moment)
- can represent various seismic sources (e.g. earthquakes, explosions)
- gives insight into the rupture *process*
- gives information about the source of the quake and its physical quantities

The **scalar seismic moment** is a measure of the strength of an earthquake representing the magnitude of the seismic moment tensor with units [Nm]. Definition of the scalar seismic moment (Kanamori and Anderson, 1975):

$$M_0 = (\tilde{L}/\tilde{C})A\Delta\sigma = \mu\bar{D}A$$

L : dimension of the fault
C :a non-dimensional shape factor
A : fault area
Δσ : (scalar) stress drop
μ : rigidity or shear modulus
D : average slip on the fault

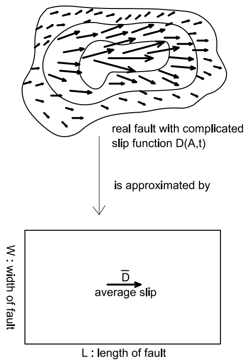


Figure 1 shows the approximation of a seismic rupture process: at the top a real rupture process is shown containing a complicated, time- and space-dependent slip function. Beneath the rupture is approximated by an average slip D on a geometrically simple fault with area A=LW. The moment is then given as product of rigidity, average slip and fault area (after Stein and Wysession, 2002)

Fig. 1

SUGGESTION OF AN ATMOSPHERIC MOMENT

Analogous to the seismic moment, a similar expression of the moment of atmospheric vortices is proposed. The connecting link between seismic and atmospheric moments is given by the stress tensor τ_{ij} in the Navier-Stokes-equation.

	Seismology	Atmosphere
Stress tensor	τ_{ij}	$\tau_{ij} = -p\delta_{ij}$ p : pressure , δ_{ij} : Kroneckerdelta
Stress drop	$\Delta\tau_{ij} = \tau^1{}_{ij} - \tau^0{}_{ij}$ $\tau^1{}_{ij}$: stress after rupture $\tau^0{}_{ij}$: initial stress	$\Delta p = p_1 - p_0$ p_0 : core pressure p_1 : environmental pressure
Moment tensor	$M^0{}_{ij} = \int dV \Delta\tau^*_{ij}$	$M^a{}_{ij} = \int dV \Delta p \delta_{ij}$
Scalar moment	$M_0 = \frac{\tilde{L}}{\tilde{C}} A\Delta\sigma$ $\Delta\sigma = \lvert\Delta\tau_{ij}\rvert$: scalar stress drop	$M_a = \frac{AL}{C}\Delta P$ A : Area of vortex, L : track length C : non-dimensional constant

V : volume ; *precisely, stress glut instead of stress drop (Backus and Mulcahy, 1976)

Interpretation of the atmospheric moment

AL has units of a volume and can also be written as (for circularly-shaped vortices): $AL=\pi R^2L=\pi R/2(LW)\sim W(LW)$, where W is the width or diameter of the vortex. LW represents the surface area affected by the vortex.
Kurgansky (2000) showed that the height of a tornado is proportional to its diameter (H/W=const.). Therefore, AL is proportional to the total affected volume V=HLW~AL (see fig.2).
C is then the constant relating height and horizontal radius, so that HLW=AL/C.
The value of C is unknown. For practical purposes it is assumed to be unity (C=1) in the next section.

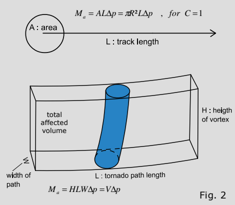


Fig. 2

Atmospheric moment of tornadoes

Assuming cyclostrophic balance and approximately constant density ($\rho\approx 1kg/m^3$), pressure deficit Δp in tornadoes is proportional to kinetic energy ($\Delta p=\rho v^2/2$). If the intensity is classified with respect to Fujita-scale, average Fujita-class velocity v is related to F-scale via (Fujita, 1971): $v(F)=6.3m/s(F+2.5)^{3/2}$, F=(0,..,5).
The atmospheric moment of tornadoes is then given by:

$$M_{a,tornadoes} = \rho ALv^2/2 \quad , \quad for \;\; C=1$$

RESULTS & CONCLUSIONS

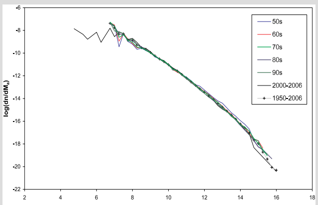
Data: US-tornado database (1950-2006), decadal analysis of probability density functions (PDFs) (Source of data : SPC, 2009)

Results: The PDFs of different periods show power-law behavior over more than 6 orders (see fig. 3):

$$dn/dM = \alpha M^{-\beta}$$

where α is a constant and β is the scaling exponent

Figure 3 shows the log-log-plot of the PDFs for different periods of tornado data. The similarity between the decades is evident just like the nearly overall linear behavior in the log-log-plot and therefore power-law behavior of the PDFs.

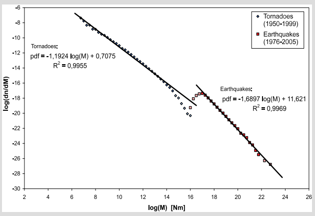


Results (cont.) : Comparison with the Gutenberg-Richter law of earthquakes
The scaling exponents of the PDFs of tornadoes and earthquakes are of similar order with slightly smaller values for tornadoes (fig. 4):

Tornadoes: $\beta_t \approx 1{,}2$

Earthquakes: $\beta_e \approx 1{,}7$

Figure 4 compares the PDFs of earthquakes and tornadoes dark blue and red squares indicate the range used for the analysis.
(source of earthquake data: The Global Centroid-Moment-Tensor Project, www.globalcmt.org, accessed: 15.09.2009)



Conclusions:

In a previous work of the authors, successful attempts have been made to find a physical parameter that unifies frequency distributions of different atmospheric low pressure systems (Schielicke and Névir, 2009). An energy of displacement has been defined, which represents the (mass-specific) work that is necessary to generate the low pressure system. Analysing frequency distributions of tornadoes and cyclones, respectively, showed exponential behavior with the same (universal) characteristic parameter with a value of approximately E_u=1000 J/kg.

The presented work gives an expansion from mass-specific to an integrated mass-related parameter (moment) that regards the system as a whole considering life-time and integrated moved mass.

OUTLOOK

- Analysis of the atmospheric moments of other vortices (e.g. cyclones)
- Investigation of the relationship between atmospheric moment and external forcing

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Lisa.Schielicke@met.fu-berlin.de
http://www.geo.fu-berlin.de/met/ag/theorie/