Incorporating Dependencies between Rainfall Statistics and Rainfall Interannual Variability into Hourly Stochastic Rainfall Generator for Improved Extreme Value Reproduction

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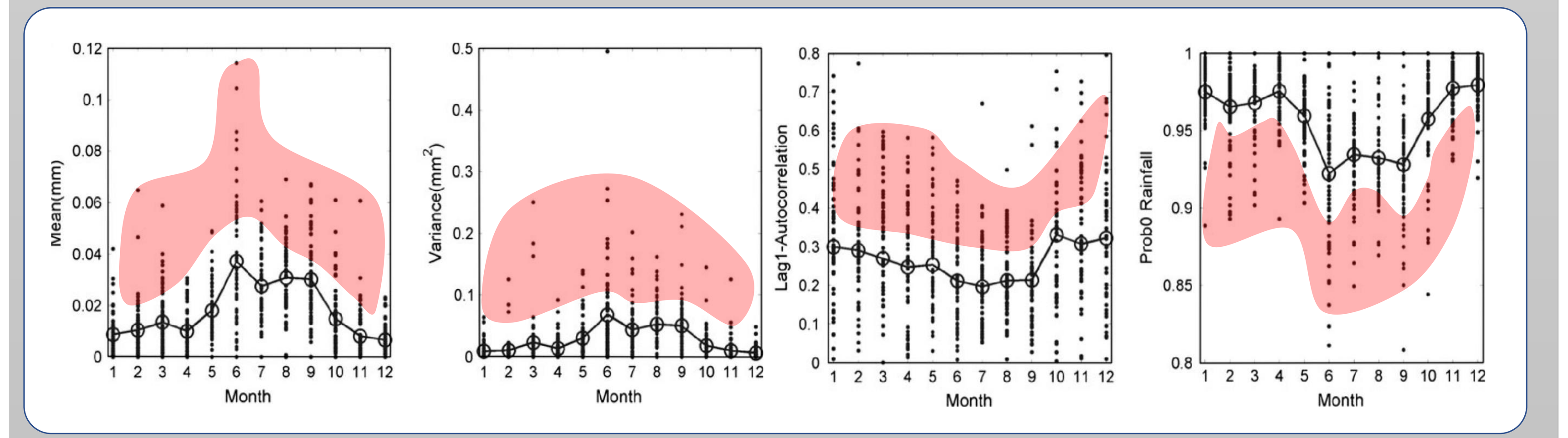
1. Introduction

- Stochastic weather generators generate time series or space-time field of synthetic weather variables of which statistics are closer to that of reality. Rainfall generator is a type of weather generator which focuses on generating synthetic rainfall data.
- Among rainfall generators, Poisson Cluster Rainfall (PCR) model is the most widely applied rainfall generator. But the traditional PCR models are known to underestimate the upper and lower extreme rainfall values.



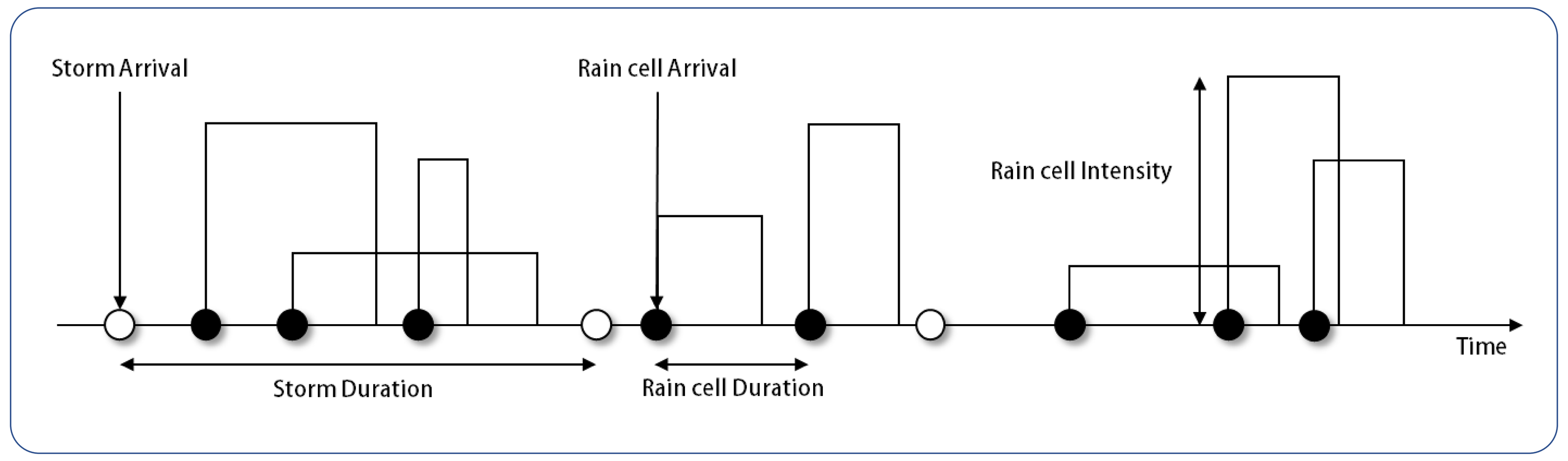
- The objective of this study is to improve the performance of PCR model to generate extreme rainfall values at hourly through daily temporal scale.
- This study employed an additional procedures to the traditional PCR model to account for the inter-annual variability.

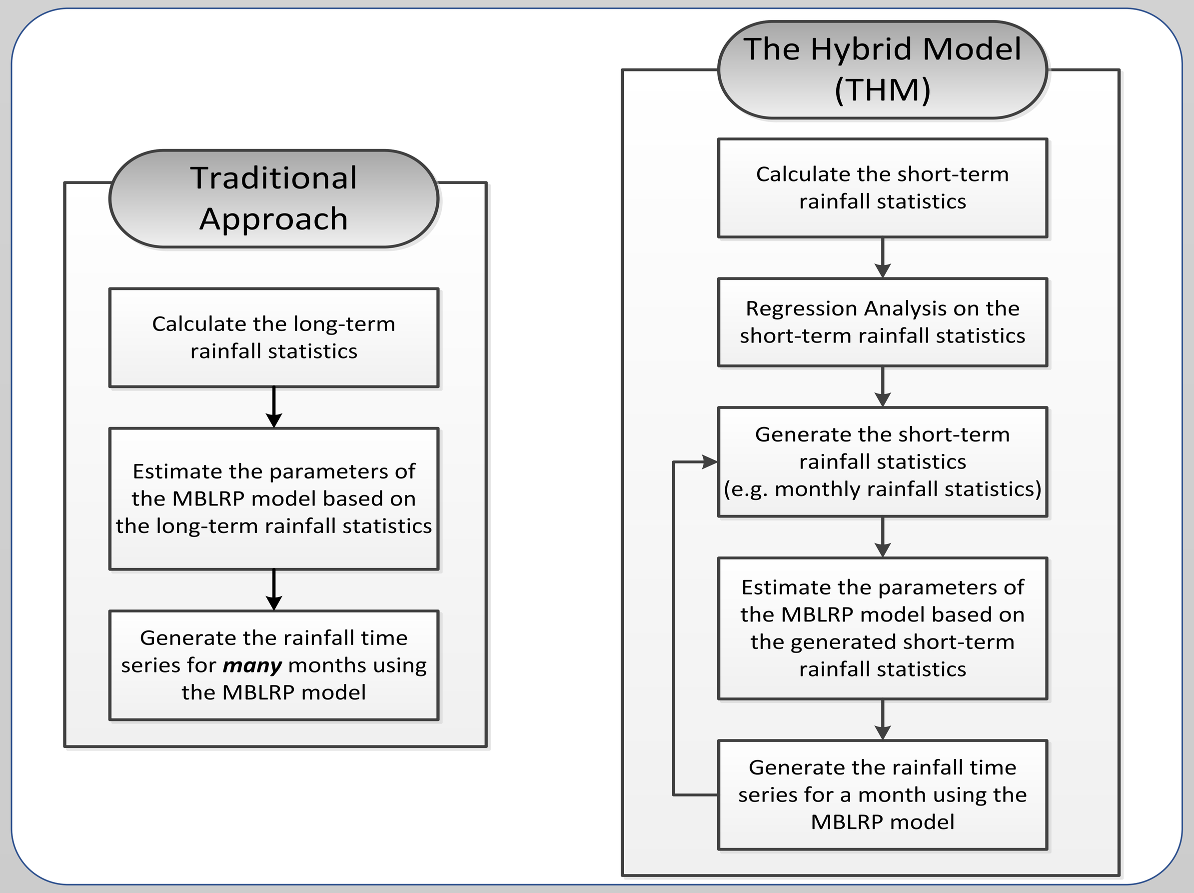
2. Study Area and Data Description

- The study area is United States and the gauges of National Climatic Data Center(NCDC) are selected .
- Among the gauges, we select the gauges which have recent observation records more than 30 years and also continuous observation.
- This study used hourly rainfall time series observed at 30 ground precipitation gages located across United States.
- This study used the data between 1981 and 2010 for model development.

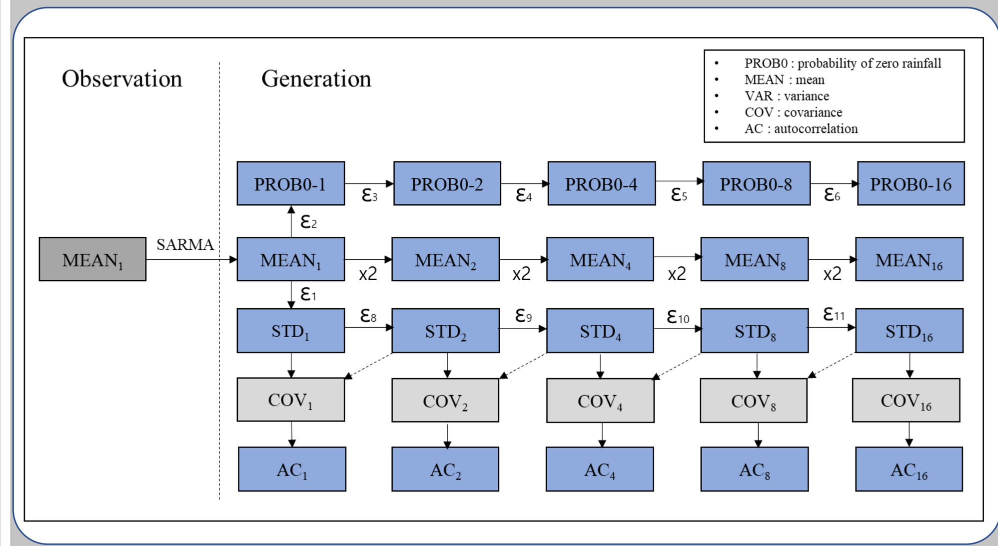
3. Methodology

- The Modified Bartlett-Lewis rectangular pulse (MBLRP) rainfall generation model was used as the hourly stochastic rainfall generator.
- The model assumes that a series of rain storms (white circles) including a series of rain cells (black circles) arrives in time according to a Poisson process.
- There are six parameters: λ, γ, β, ν, α and μ which can make synthetic rainfall time series.
- It is customary to use the ratios ϕ = γ/η and κ = β/η as parameters instead of γ and β





- The model presented in this study incorporates the inter-annual variability of rainfall statistics by simulating rainfall statistics of individual months based on the correlation between observed rainfall statistics.
- This model is named The Hybrid Model (THM) because it combines the process of generating rainfall statistics and the process of generating rainfall time series.
- The six parameters of the MBLRP model were calibrated such that the statistics of the synthetically generated rainfall time series resemble those of the observed time series.
- This study used 4 kinds of statistics at 1-, 2-, 4-, 8- and 16-hourly accumulation levels($4 \times 5 = 20$).
- For generation of statistics, hourly mean of the rainfall time series of a given calendar month for the current period was drawn based on Seasonal ARMA(SARMA) model.
- Especially, the process of generating AC was changed to consider correlation with generated variance and also other statistics were generated through multivariate distribution between errors of observed statistics.



Estimation of Autocorrelation(AC)

- AC_{OBS}

$$=> VAR(X_1+X_2) = VAR(X_1) + VAR(X_2) + 2Cov(X_1,X_2)$$

$$=> \frac{VAR(X_1+X_2)-VAR(X_1)-VAR(X_2)}{2} = Cov(X_1,X_2)$$

$$=> \frac{VAR(X_1+X_2)-VAR(X_1)-VAR(X_2)}{2*VAR(X)} = \frac{Cov(X_1,X_2)}{VAR(X)} = AC_{OBS}$$

- AC_{EST}

$$=> \frac{VAR(2X)-2*VAR(X)}{2*VAR(X)} = \frac{1}{2}\frac{VAR2}{VAR1} - 1 = AC_{EST}$$

$$\frac{VAR(X_1+X_2)-VAR(X_1)-VAR(X_2)}{2*VAR(X)} \cong \frac{VAR(2X)-2*VAR(X)}{2*VAR(X)}$$

- The parameters were estimated such that the difference between the statistics of the observed and that of the synthetic rainfall time series are minimized.
- Estimated parameters were used for generating synthetic rainfall time series through MBLRP model.

$$E\left[Y_t^{(T)}\right] = \lambda\mu\mu_c\frac{\nu}{\alpha-1}T$$

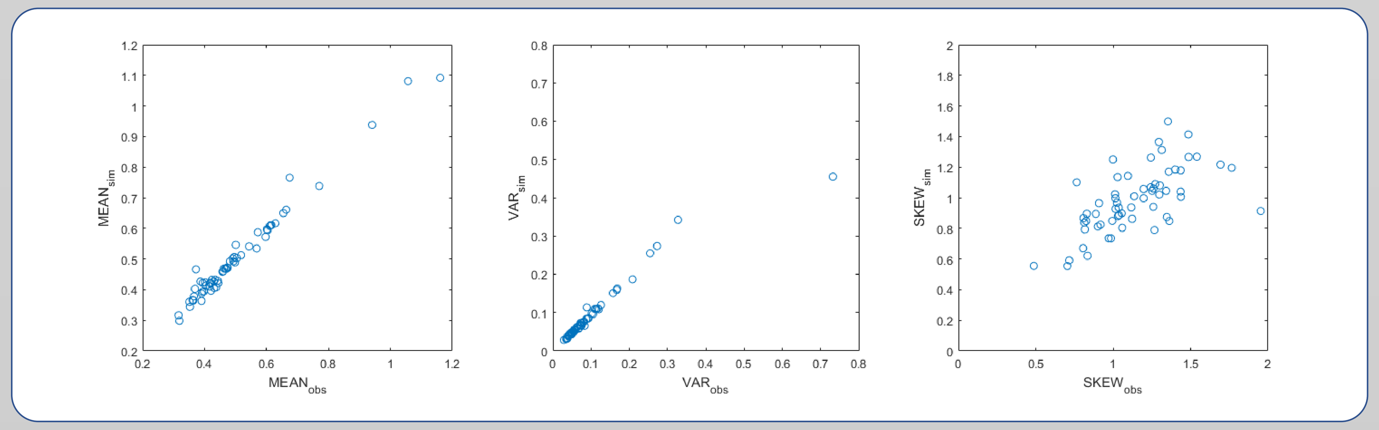
$$\mathrm{Var}\left[Y_t^{(t)}\right] = \frac{2\nu^{2-\alpha}T}{\alpha-2}\left(k_1-\frac{k_2}{\phi}\right) - \frac{2\nu^{3-\alpha}}{(\alpha-2)(\alpha-3)}\left(k_1-\frac{k_2}{\phi^2}\right) + \frac{2}{(\alpha-2)(\alpha-3)}\left[k_1(T+\nu)^{3-\alpha} - \frac{k_2}{\phi^2}(\phi T+\nu)^{3-\alpha}\right]$$

$$\mathrm{Cov}\left[Y_t^{(T)},Y_{t+s}^{(T)}\right] = \frac{k_1}{(\alpha-2)(\alpha-3)}\{[T(s-1)+\nu]^{3-\alpha} + [T(s+1)+\nu]^{3-\alpha} - 2(Ts+\nu)^{3-\alpha}\} + \frac{k_2}{\phi^2(\alpha-2)(\alpha-3)}\{2(\phi Ts+\nu)^{3-\alpha} - [\phi T(s-1)+\nu)^{3-\alpha} - [\phi T(s+1)+\nu]^{3-\alpha}\}$$

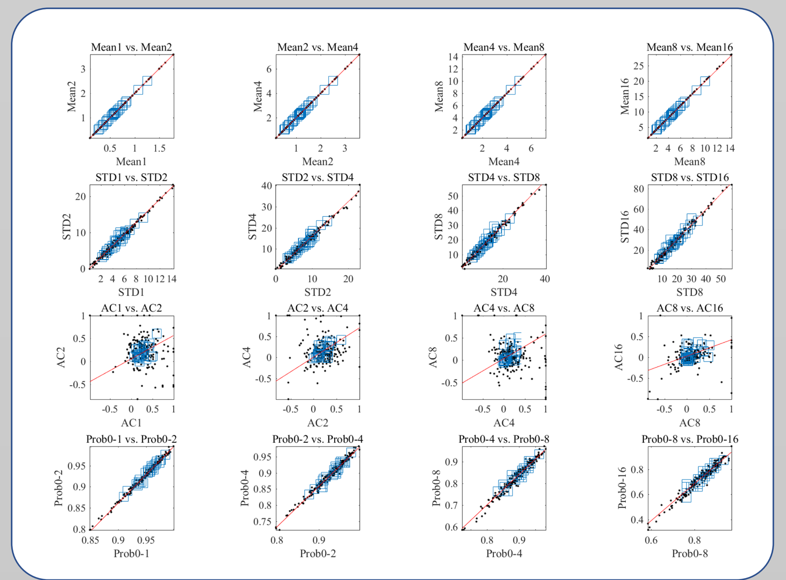
$$P(\text{zero rainfall}) = \exp\Big\{-\lambda T - \frac{\lambda\nu}{\phi(\alpha-1)}\left[1+\phi(\kappa+\phi) - \frac{1}{4}\phi(\kappa+\phi)(\kappa+4\phi) + \frac{\phi(\kappa+\phi)(4\kappa^2+27\kappa\phi+72\phi^2)}{72}\right] + \frac{\lambda\nu}{(\alpha-1)(\kappa+4\phi)}\left(1-\kappa-\phi+\frac{3}{2}\kappa\phi+\phi^2+\frac{\kappa^2}{2}\right) + \frac{\lambda\nu}{(\alpha-1)(\kappa+\phi)}\left[\frac{\nu}{\nu+(\kappa+\phi)T}\right]^{\alpha-1}\frac{\kappa}{\phi}\left(1-\kappa-\phi+\frac{3}{2}\kappa\phi+\phi^2+\frac{\kappa^2}{2}\right)\Big\}$$

$$k_1 = \left(2\lambda\mu_c\mu^2 + \frac{\lambda\mu_c\kappa\phi\mu^2}{\phi^2-1}\right)\left(\frac{\nu^\alpha}{\alpha-1}\right),\ k_2 = \left(\frac{\lambda\mu_c\kappa\phi\mu^2}{\phi^2-1}\right)\left(\frac{\nu^\alpha}{\alpha-1}\right),\ \mu_c = 1+\frac{\kappa}{\phi}$$

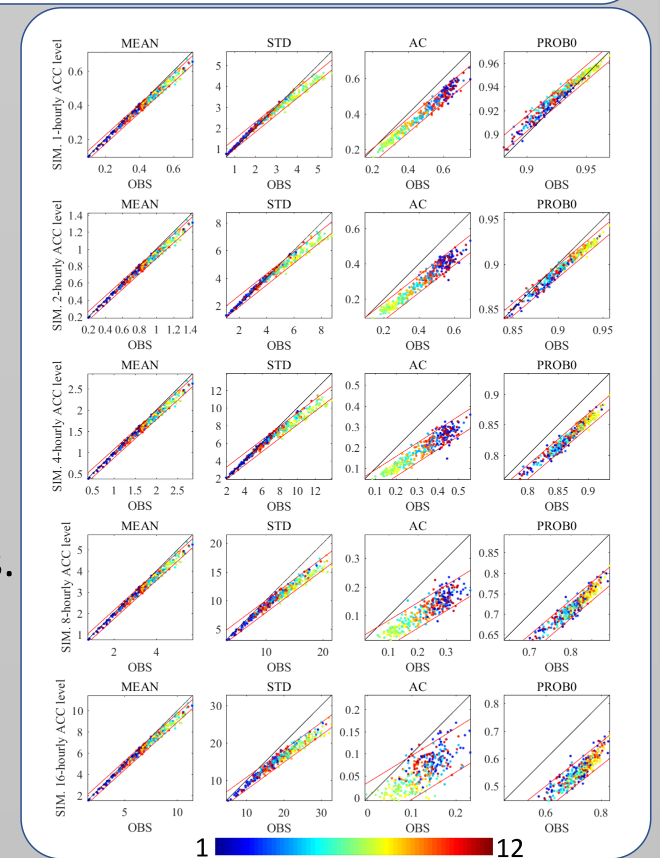
4. Results and Conclusion



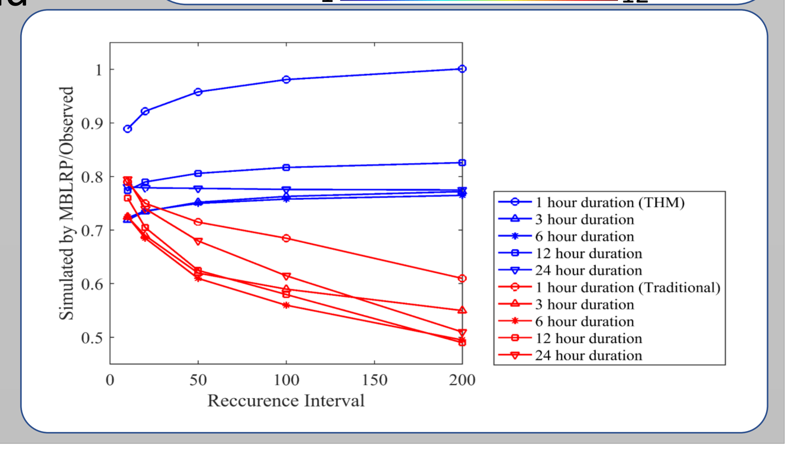
- This is the generation result of monthly mean from SARMA model. As shown in this figure, generated monthly means have similar statistical characteristics with observed mean.
- This is 200 sets of generated statistics and observed statistics. The method estimating autocorrelation improve reproducing performance of statistics compared to last methodology.



- Right figure compares 4 statistics of the observed rainfall and 200 years of synthetically generated rainfall.
- Rainfall mean and standard deviation shows good match while lag1-autocorrelation (AC) and probability of zero rainfall (PROB0) does not.
- Latter two variables are more related to intra- and inter- storm characteristics of rainfall which is too subtle to perfectly express using only 6 parameters.



- The figure shows the slope of the zero-intercept regression line comparing the design rainfall estimated based on the observed rainfall and the one estimated based on this study.
- The degree of design rainfall underestimation of the rainfall generation model were larger at the traditional PCR model than THM in the most of cases.



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Stochastic Weather Generators for Hydrological Applications

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