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Picture: Huw Alexander Ogilvie

Sub-kilometer-scale space-time stochastic rainfall simulation

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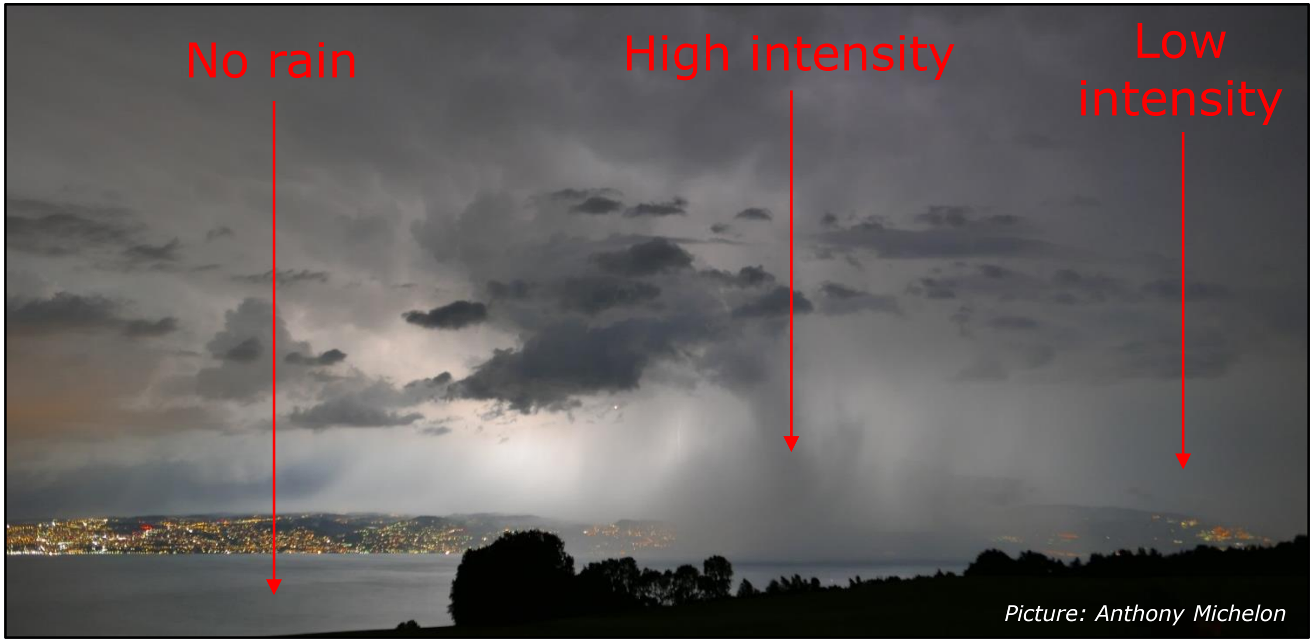
| le savoir vivant |

SWGen-Hydro 2017 – Berlin - 20 September 2017

Introduction

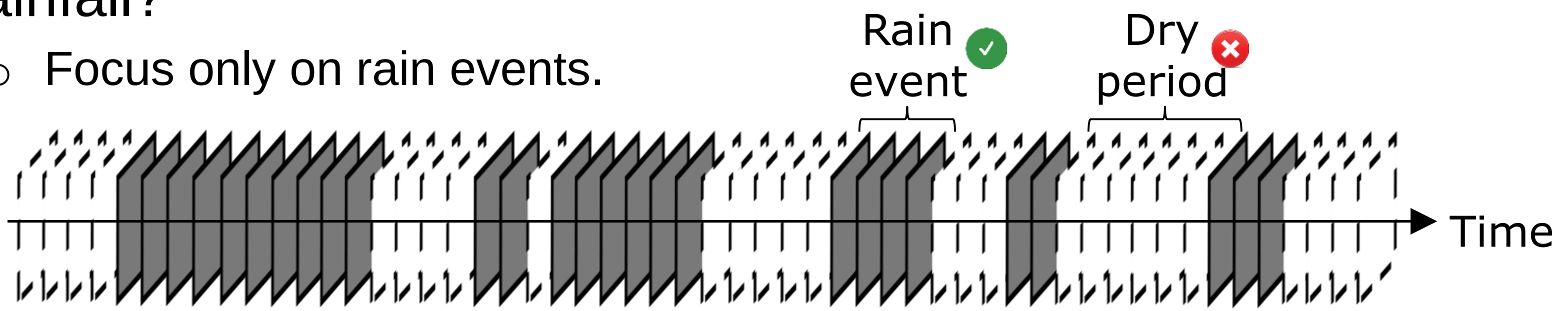
- When observed at the local scale, rainfall appears highly variable in space and in time within rain events.



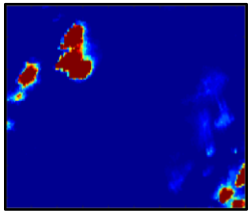
=> How can we measure and reproduce the statistical behavior of rainfall?

Introduction

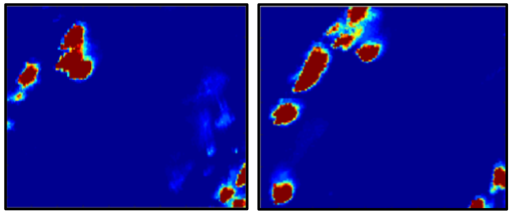
- How can we measure and reproduce the statistical behavior of rainfall?
  - Focus only on rain events.



  - Within rain events, pay a particular attention to space-time dependencies:



- ➢ Spatial correlation?
- ➢ Spatial patterns?
- ➢ Nature of rain / no-rain transitions?



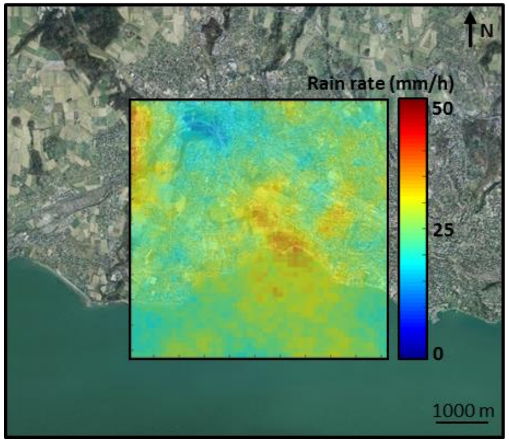
- ➢ Temporal correlation?
- ➢ Morphing of spatial patterns?
- ➢ Advection of rain storm?

Introduction

- How can we measure and reproduce the statistical behavior of rainfall?

=> Use a stochastic rainfall model.

  - Input data: rain rate time series (high resolution rain gauges).
  - Parameter inference:
    - Calibration of the model.
    - The calibrated model gives insights on the structure of rainfall.
  - Simulation: generate synthetic rain fields which reproduce the structure of observed rain fields.



Instruments

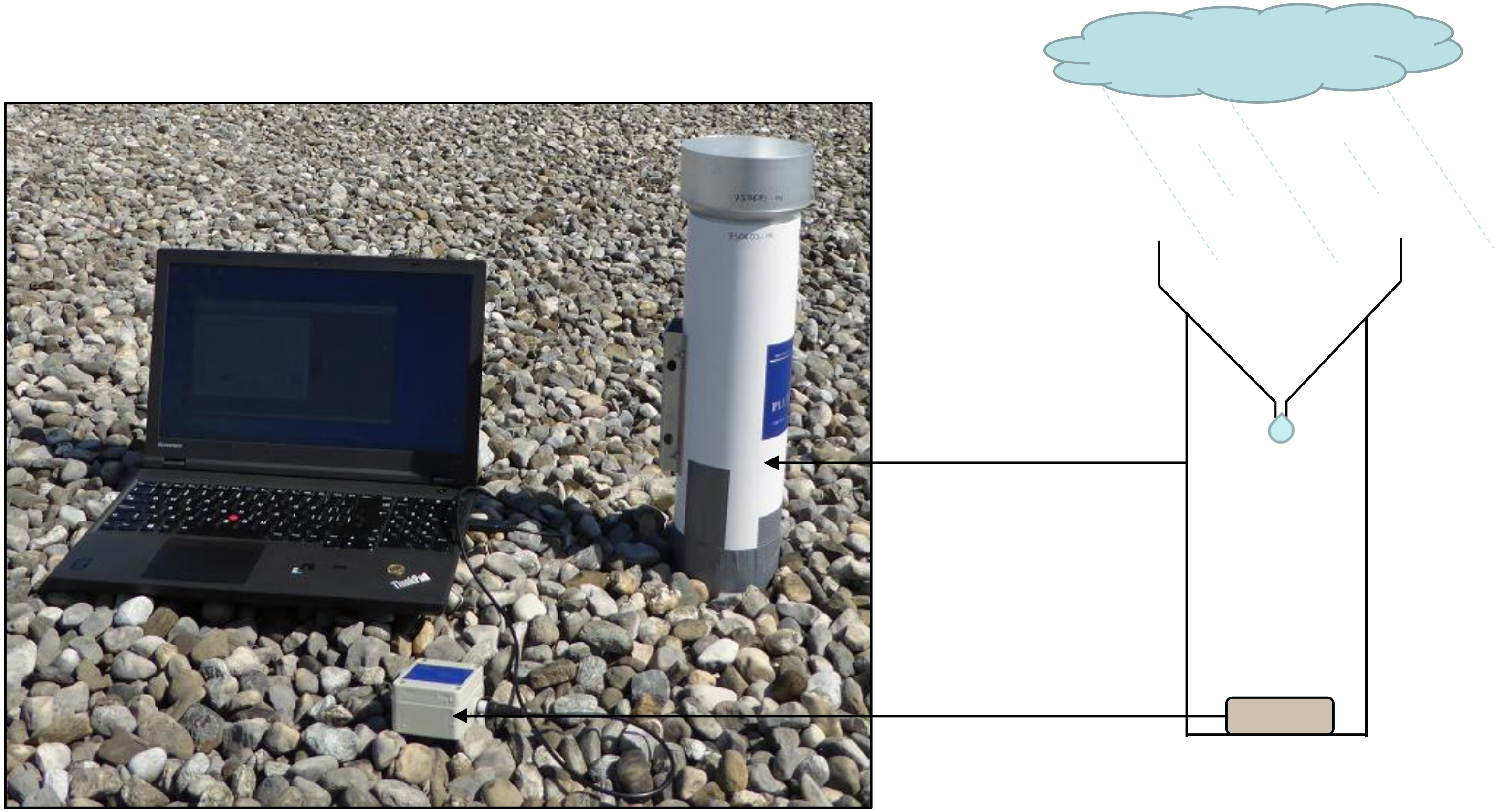
Questions: - How to measure local rain fields?

- Which data to feed a local scale stochastic rainfall model?

Instruments

Drop counting rain gauge

- Drop counting rain gauge (Pluvimate).
- Operation principle: counts calibrated drops instead of bucket tips.

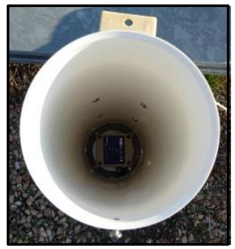


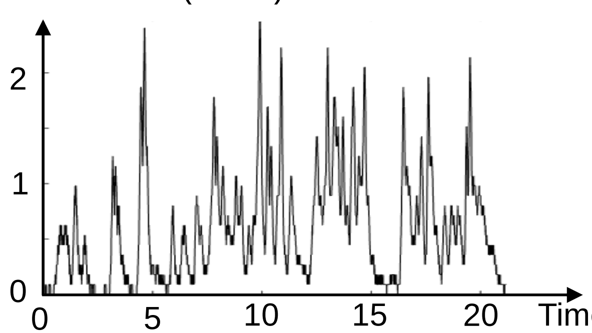
Instruments

High resolution

Pluvimate

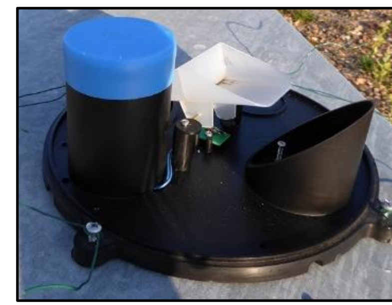
- Operation principle: Drop counting.
- Rain height resolution: 0.01mm.
- Sampling rate: 30sec – 1min.

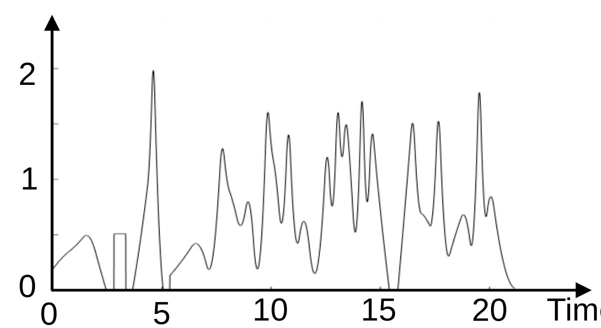




Watchdog 1120

- Operation principle: Tipping bucket.
- Rain height resolution: 0.1-0.25mm.
- Sampling rate: 5 – 10 min.



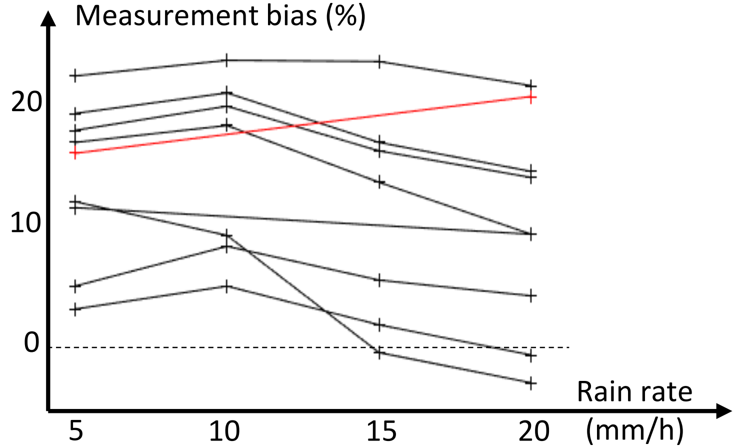


Instruments

Calibration

Absolute calibration

Artificial rain, 0-20mm/h

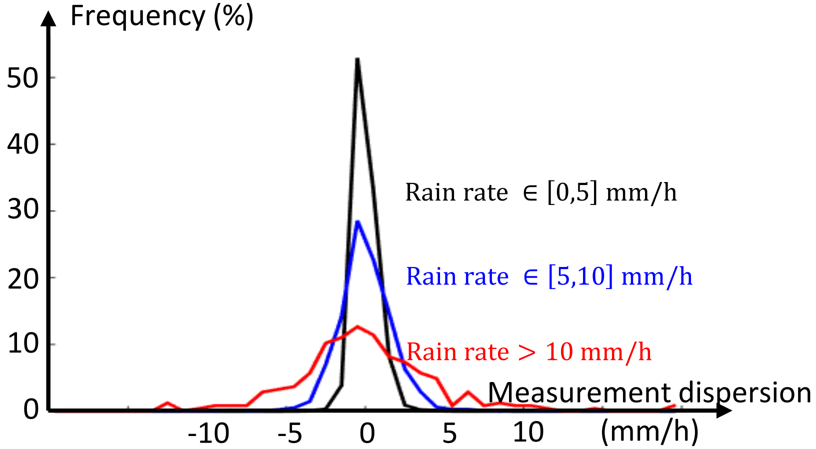


Black: Pluvimate device

Red: Tipping bucket rain gauge (for comparison)

Relative calibration

Natural rain, 0-40mm/h



Instruments

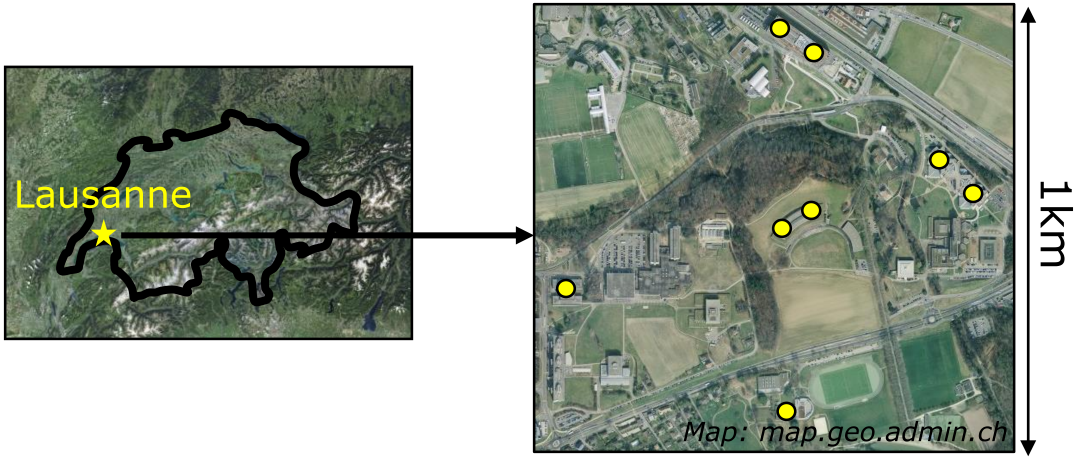
Low cost and easy to set up

- Relatively cheap (~500$ per device).
- Easy to set up (light, no moving parts, low power consumption).
- Few maintenance (except low storage capacity).

=> Dense networks with many gauges, even in mountains.

Instruments

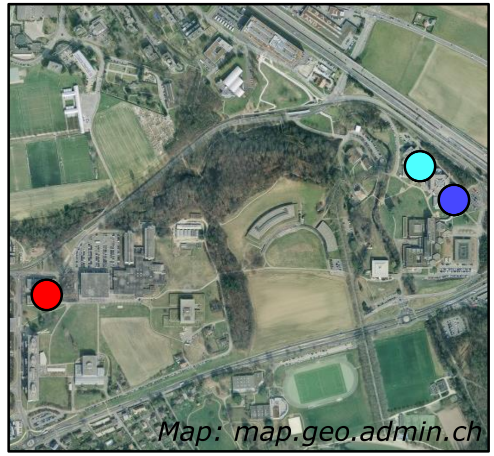
Experimental network



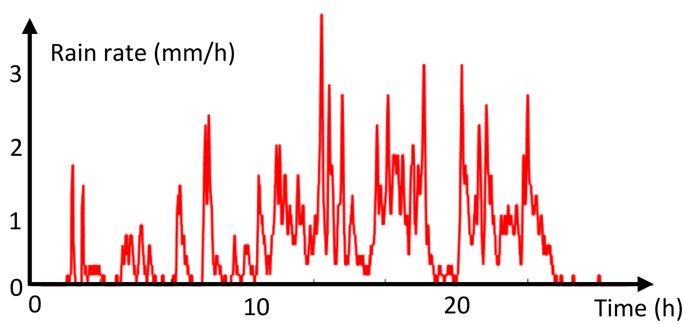
- 8 rain gauges.
- 1km x 1km.
- Sampling rate: 30sec.

Instruments

Observed rainfall structure

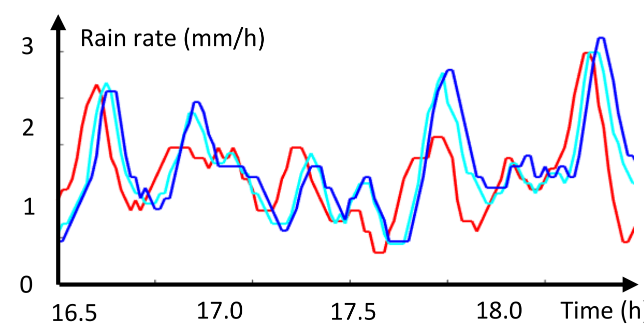


Single site observations



- Mass of zero measurements.
- Smooth dry/wet transitions.

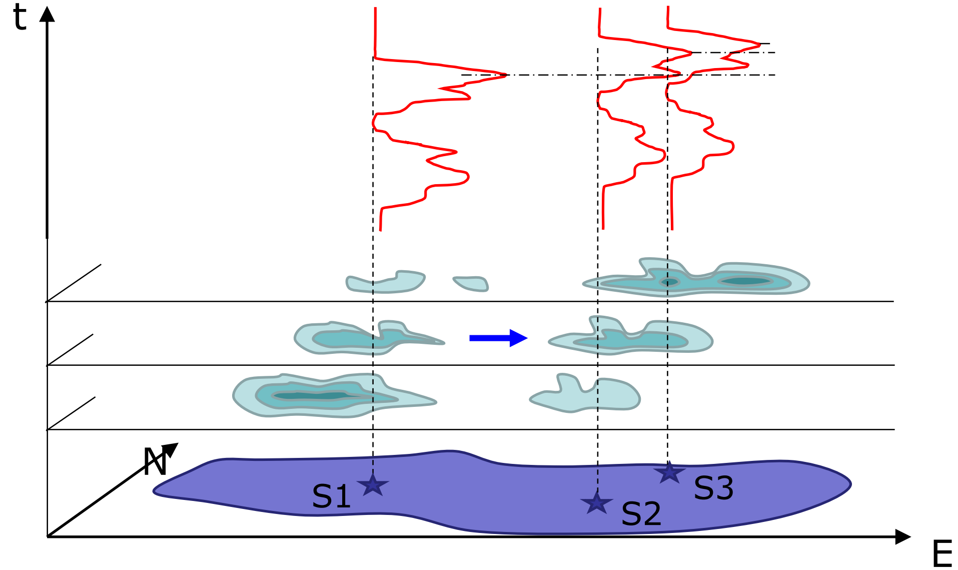
Multi-sites observations



- Spatial variability of time series.
- More similarities for close gauges.
- Time shift for distant gauges.

Stochastic rainfall model

Question: How to choose a stochastic model which can handle the features of rainfall arising from Pluvimate data?



Observed structure

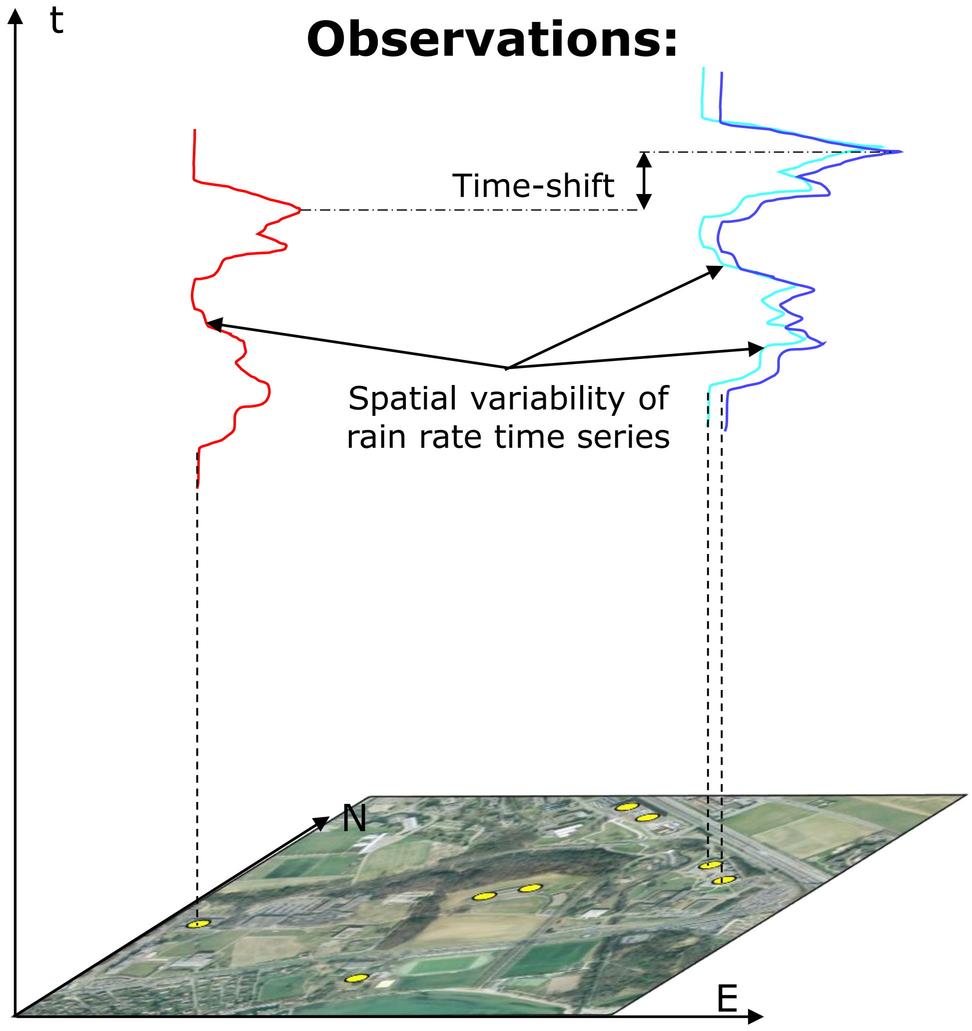
Conceptual model

$$R_m = \mathrm{f}(\theta)$$

Parametrization

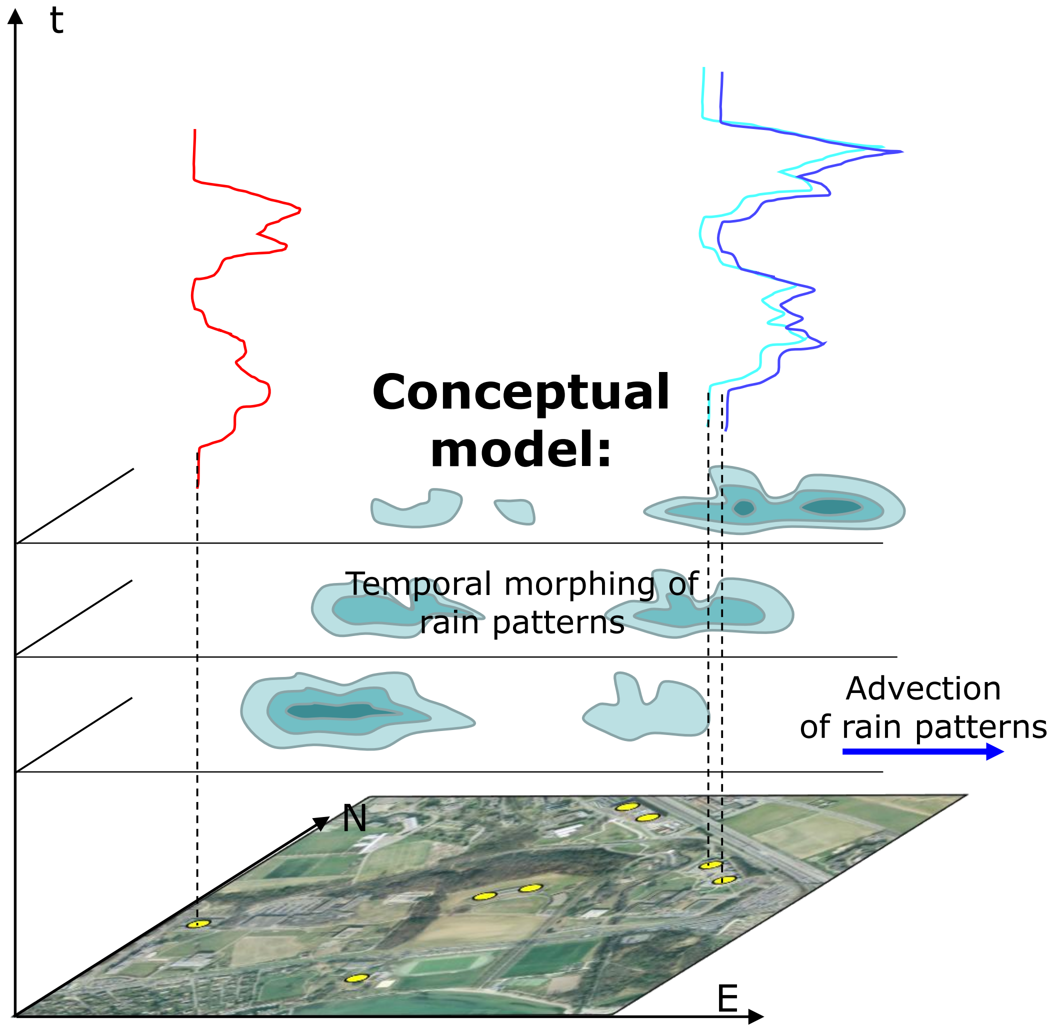
Stochastic rainfall model

Conceptual model



Stochastic rainfall model

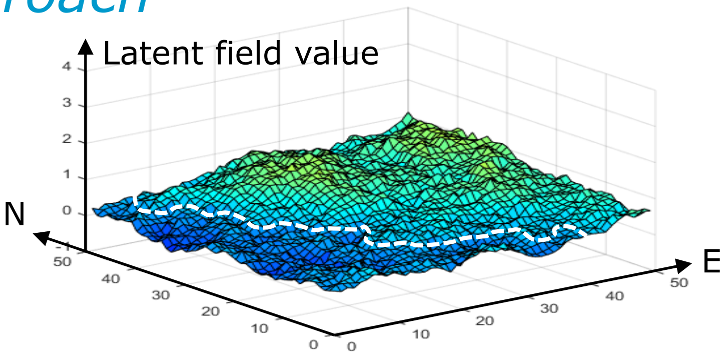
Conceptual model



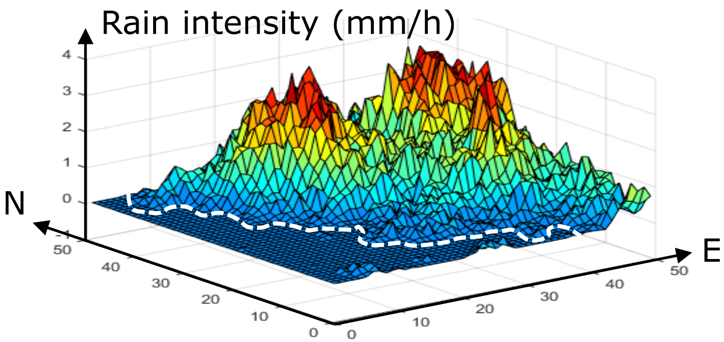
Stochastic rainfall model

Overall modelling approach

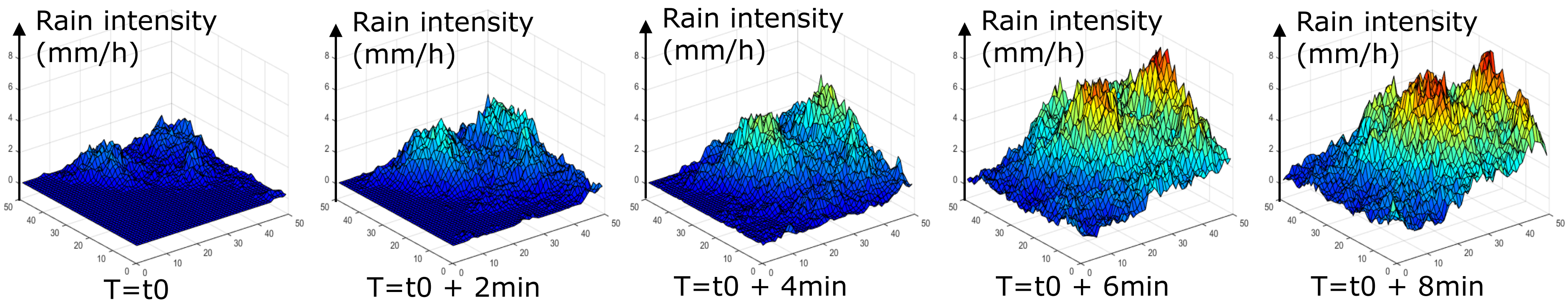
- Latent Gaussian random field.



- Precipitation arise from the latent field through truncation and transformation.



- Rainfall dynamics is modeled by an asymmetric and non-separable covariance function.



Stochastic rainfall model

Parametrization

- Latent Gaussian random field:
  - Spatio-temporal coordinates: $(\boldsymbol{s}, t)$.
  - Standardized and stationary multivariate Gaussian random field: $Y(\boldsymbol{s}, t)$.

- Precipitation arise from the latent field through truncation and transformation:

$$\begin{cases} R_m(\boldsymbol{s}, t) = 0 & if\ Y(\boldsymbol{s}, t) \leq a_0 \\ R_m(\boldsymbol{s}, t) = \left(\frac{Y(\boldsymbol{s}, t) - a_0}{a_1}\right)^{\frac{1}{a_2}} & if\ Y(\boldsymbol{s}, t) > a_0 \end{cases}$$

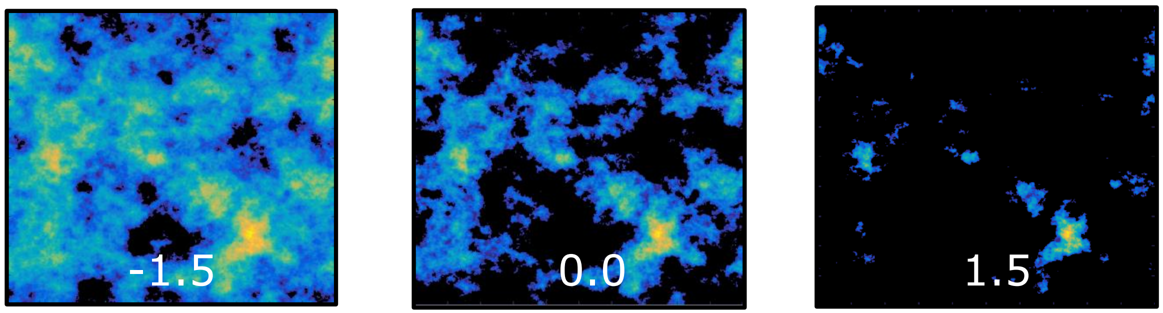
- Rainfall dynamics is modeled by an asymmetric and non-separable covariance function $\rho(\boldsymbol{s}, t)$:
  - Advection is modeled by a single vector V: $\rho(d\boldsymbol{s} - V.dt, dt) = \rho_L(d\boldsymbol{s}, dt)$.
  - Diffusion / morphing is modelled by a non-separable covariance in a Lagrangian reference frame:

$$\rho_L(d\boldsymbol{s}, dt) = \frac{1}{\left(\left(\frac{dt}{d}\right)^{2\delta} + 1\right)} . \exp\left(- \frac{\left(\frac{d\boldsymbol{s}}{c}\right)^{2\gamma}}{\left(\left(\frac{dt}{d}\right)^{2\delta} + 1\right)^{\beta\gamma}} \right)$$

Stochastic rainfall model

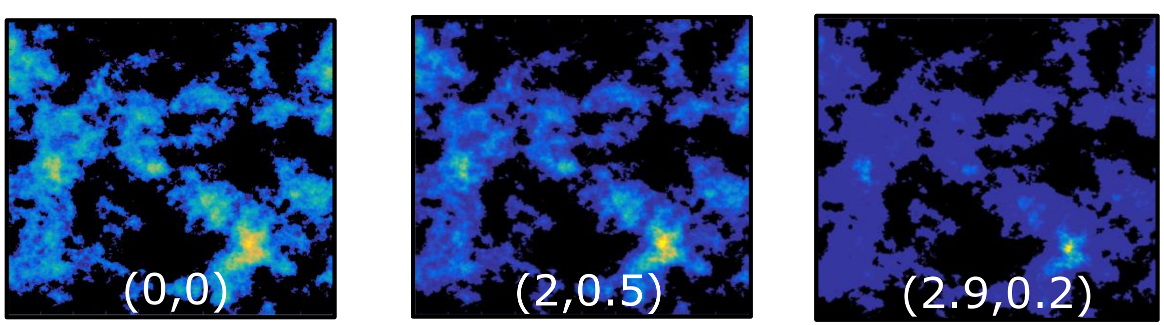
Parametrization

- Transform function:
  - Truncation => proportion of dry areas: $R_m(\boldsymbol{s},t) = 0 \qquad if\ Y(\boldsymbol{s},t) \leq a_0$



  - Transform function => skewness of the marginal distribution:

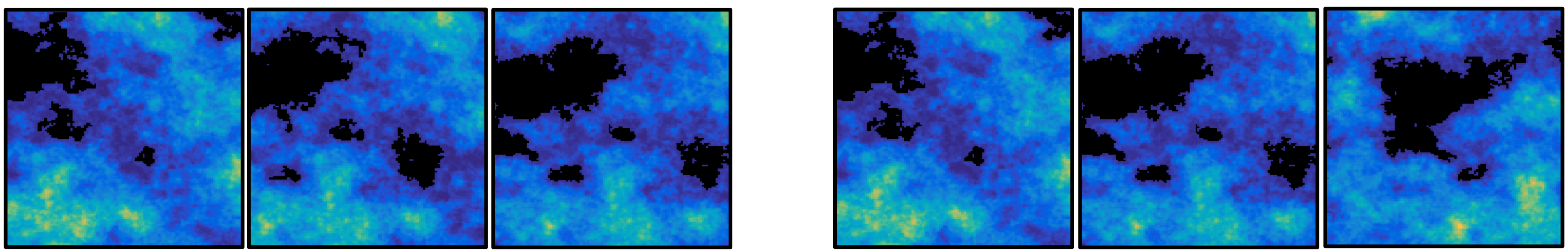
$$R_m(\boldsymbol{s},t) = \left(\frac{Y(\boldsymbol{s},t) - a_0}{a_1}\right)^{1/a_2} \quad if\ Y(\boldsymbol{s},t) > a_0$$



Stochastic rainfall model

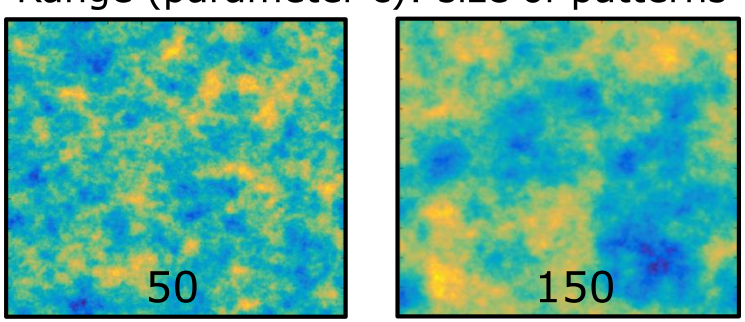
Parametrization

- Covariance of the latent field:
  - Advection vector: $\rho(d\mathbf{s} - V.dt, dt) = \rho_L(d\mathbf{s}, dt)$.

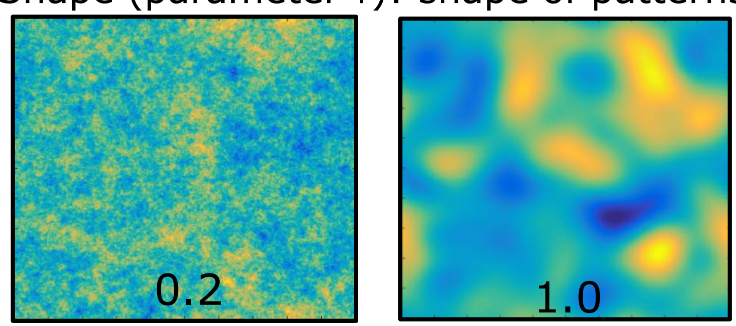


  - Spatial dependencies: $\rho_L(d\mathbf{s}, dt) = \frac{1}{\left(\left(\frac{dt}{d}\right)^{2\delta} + 1\right)} . \exp\left(- \frac{\left(\frac{d\mathbf{s}}{c}\right)^{2\gamma}}{\left(\left(\frac{dt}{d}\right)^{2\delta} + 1\right)^{\beta\gamma}} \right)$

Range (parameter c): size of patterns



Shape (parameter ϒ): shape of patterns



Stochastic rainfall model

Parameter inference

- The inference of model parameters allows to:
  - Calibrate the model.
  - Gain insights on the structure of rainfall.
- A Bayesian approach is selected to account for the uncertainty on model parameters.
  - Use a Metropolis-Hastings sampler, which requires:
    - A statistical model to calibrate => stochastic rainfall model.
    - A calibration dataset => Rain rate time series.
    - The likelihood of observations given model parameters => Easy to derive thanks to the assumption of multivariate Gaussian latent field.

Metropolis-Hasting algorithm

1) Initialize model parameter $\theta \in \Theta$
2) (a) Generate $\theta^* \sim q(\theta^*|\theta)$ and $u \sim U_{[0,1]}$ (q = proposition kernel)

   (b) If $u < \min\left(1, \frac{l(R_m|\theta^*) \times q(\theta|\theta^*)}{l(R_m|\theta\ ) \times q(\theta^*|\theta)}\right)$, then $\theta = \theta^*$
3) Iterate 2)

Stochastic rainfall model

Parameter inference

- For large datasets, the calculation of the full-likelihood is computationally infeasible.

$$L(R_m|\theta) = -0.5[\log|\Sigma_{++}| - Z_+^t.\Sigma_{++}^{-1}.Z_+ - N_+.\log(2\pi)] \longrightarrow |.| \; and \; .^{-1} \; for \; large \; matrices$$

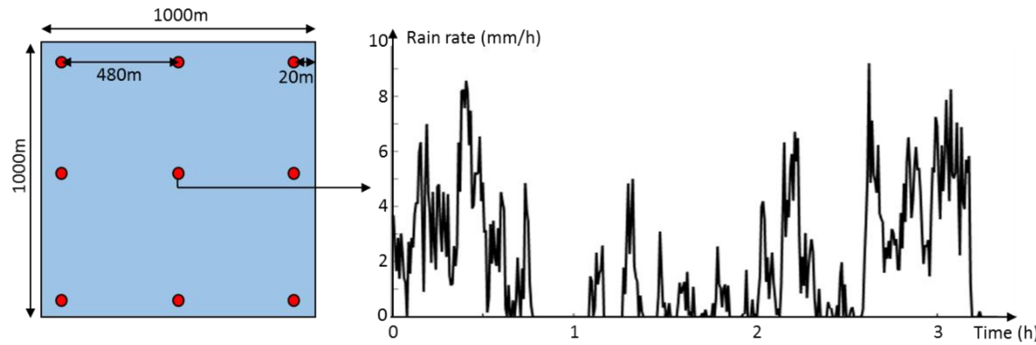
$$+\log \phi_{N_0}(a_0|\Sigma_{+0}\Sigma_{++}^{-1}, \Sigma_{00} + \Sigma_{+0}\Sigma_{++}^{-1}\Sigma_{0+}) \longrightarrow joint \; cdf \; \phi_{N_0} \; at \; many \; points$$

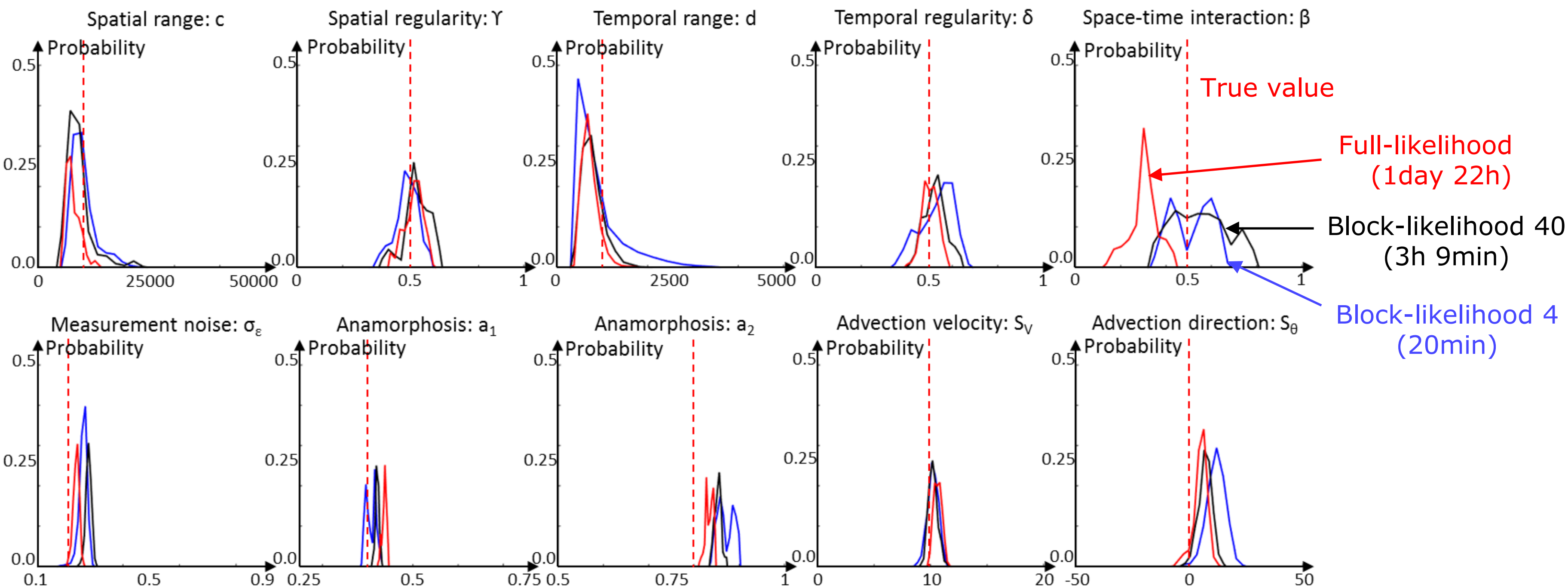
- Approximations are required:
  - For positive measurements, the likelihood is evaluated for small blocks.
    - Blockwise likelihood: $l(\theta|R_{I_+}) \approx \prod_{p=1}^{N_\tau} l(\theta|R_{BI_+})$
  - For zero measurements, the censored values of the latent field are simulated by a Gibbs sampler within the Metropolis-Hasting algorithm.

Stochastic rainfall model

Parameter inference

- In practice, the previous approximations don't worsen the inference of parameters:
  - Test on a synthetic case:





Stochastic rainfall model

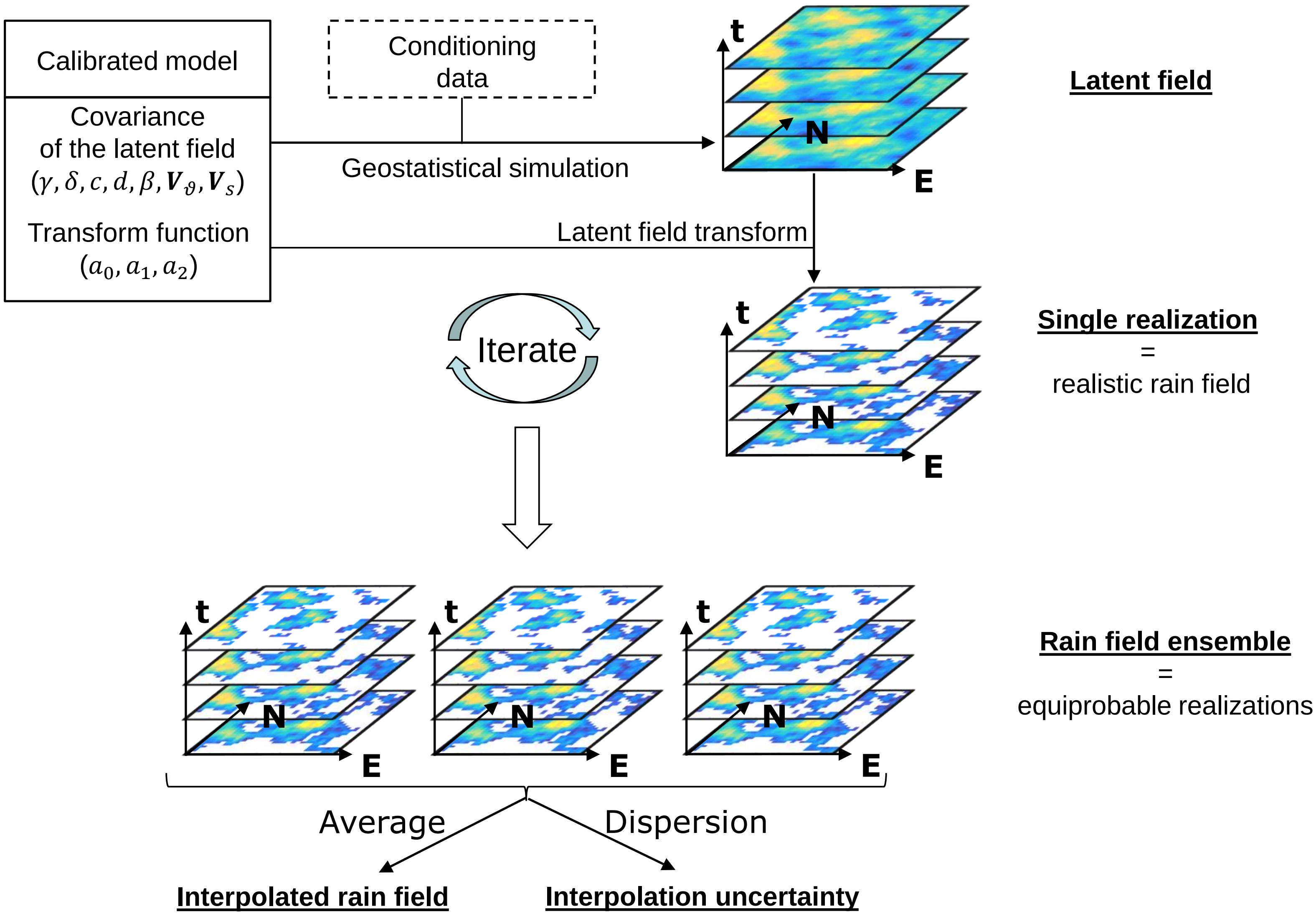
Simulation

- The posterior distribution of the stochastic rainfall model can be used to generate synthetic rain fields, which are useful to:
  - Simulate rainfall over the space-time domain of interest.
  - Interpolate rain at any ungauged location (or time step):
    - Predicted value.
    - Assessment of prediction error.

- Simulation method:
  - The latent field is first obtained by geostatistical simulation.
  - Synthetic rain fields are derived by censoring & transforming the latent field.

Stochastic rainfall model

Simulation



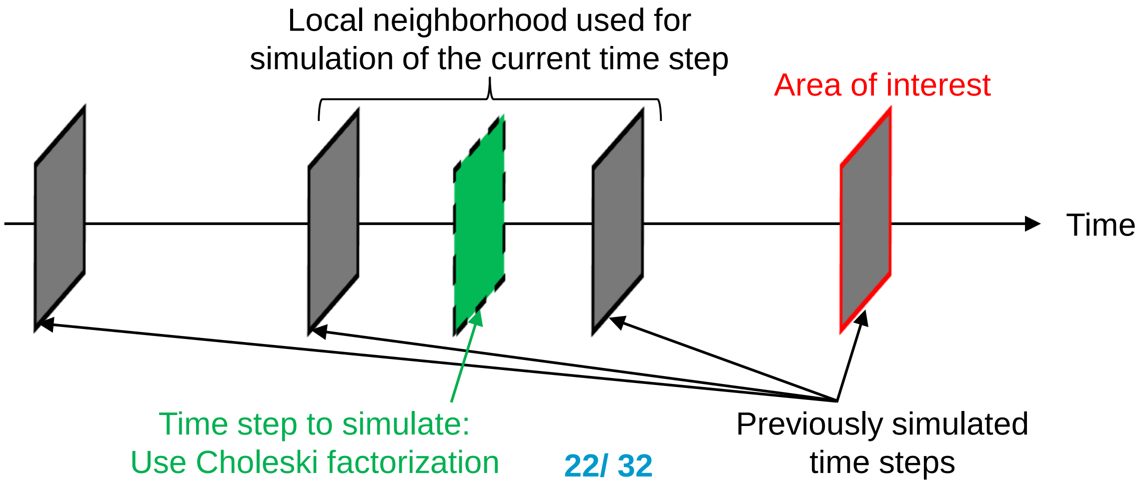
Stochastic rainfall model

Simulation

- For large space-time simulation grids, exact geostatistical simulations of the latent field are computationally unfeasible.
  - Require Choleski factorization of large covariance matrices.

- Usual fast simulation methods (turning bands, FFT-based, etc.) cannot be easily applied in the current context.

- An ad-hoc simulation method has been developed:
  - Choleski factorization in the space dimension.
  - Multigrid Sequential Gaussian Simulation (SGS) in the time dimension.



Stochastic rainfall model

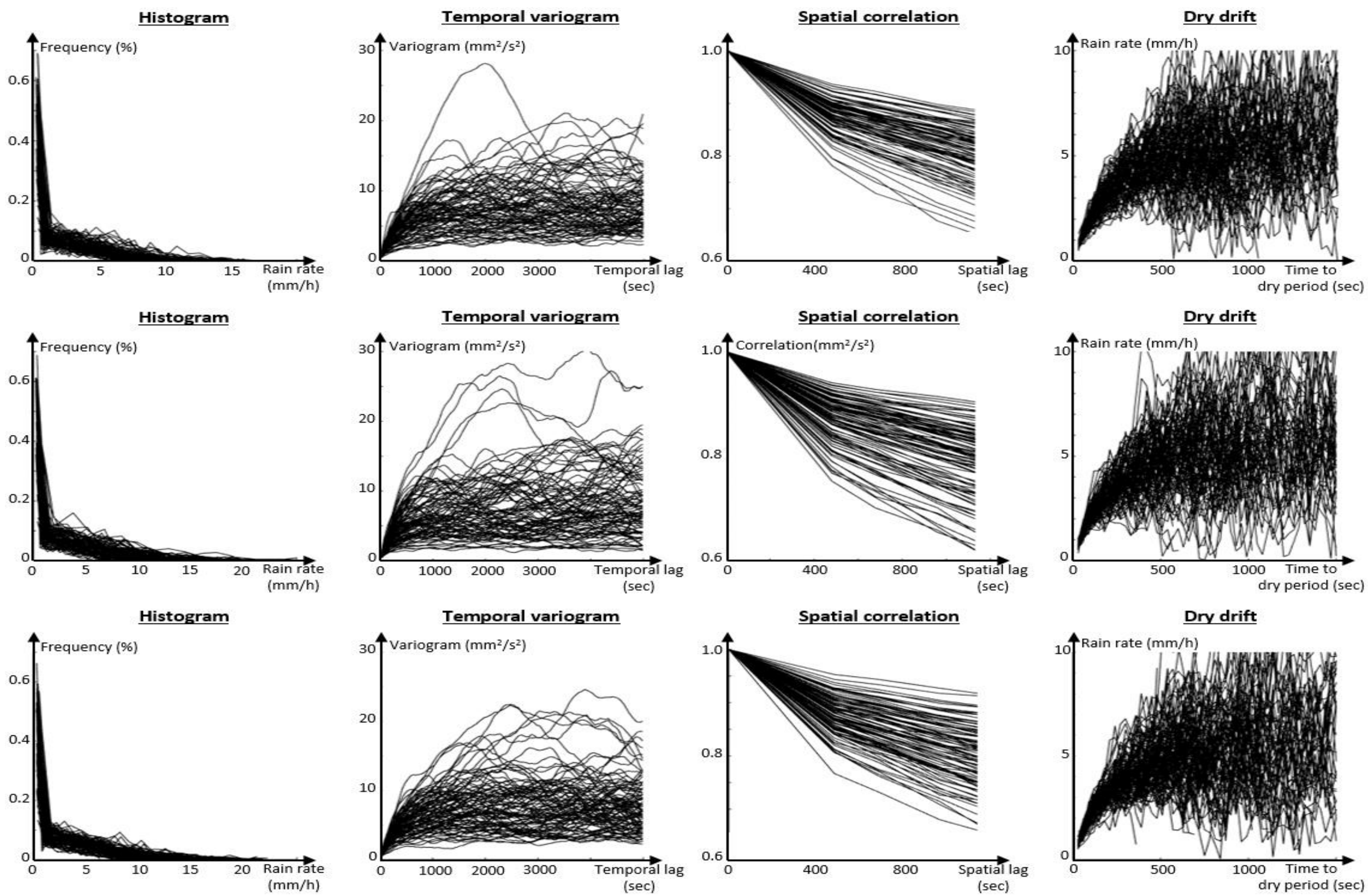
Simulation

- In practice, the proposed simulation method does not worsen the reproduction of rainfall statistics.

Exact simulation (Choleski decomposition)

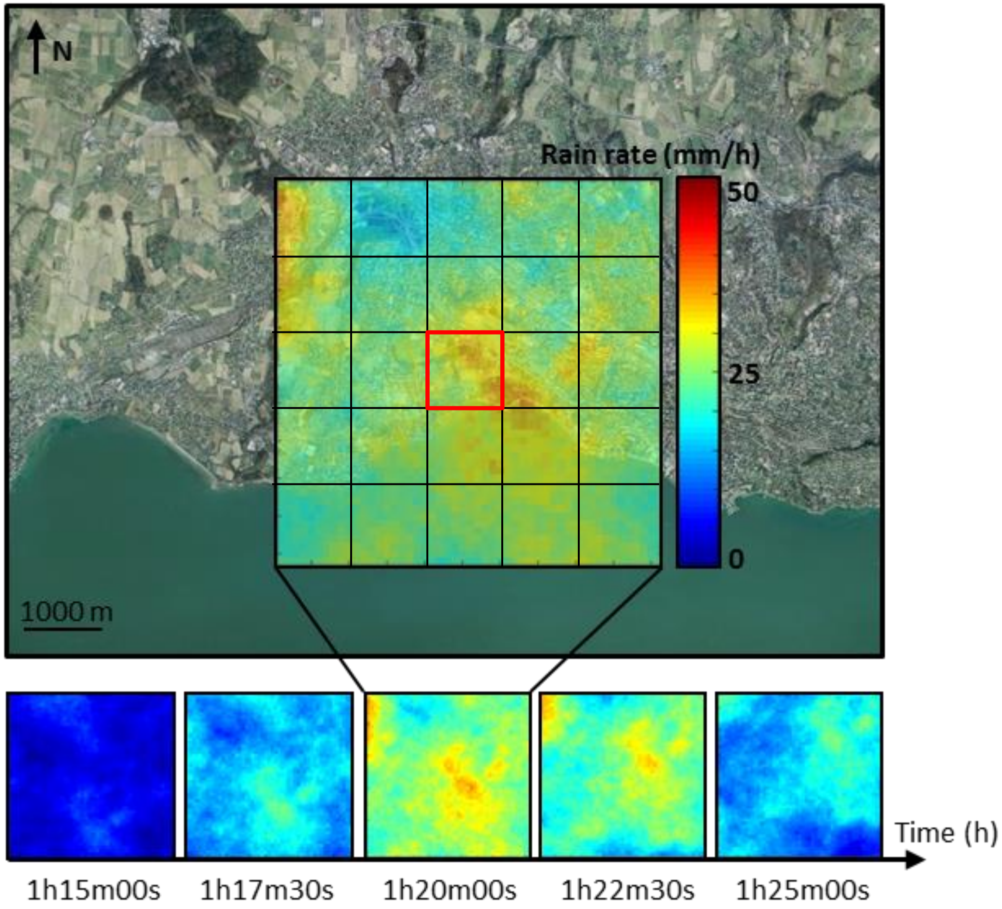
Approximated simulation (40 neighboring time steps)

Approximated simulation (4 neighboring time steps)



Application

Question: What happens within a single radar pixel?

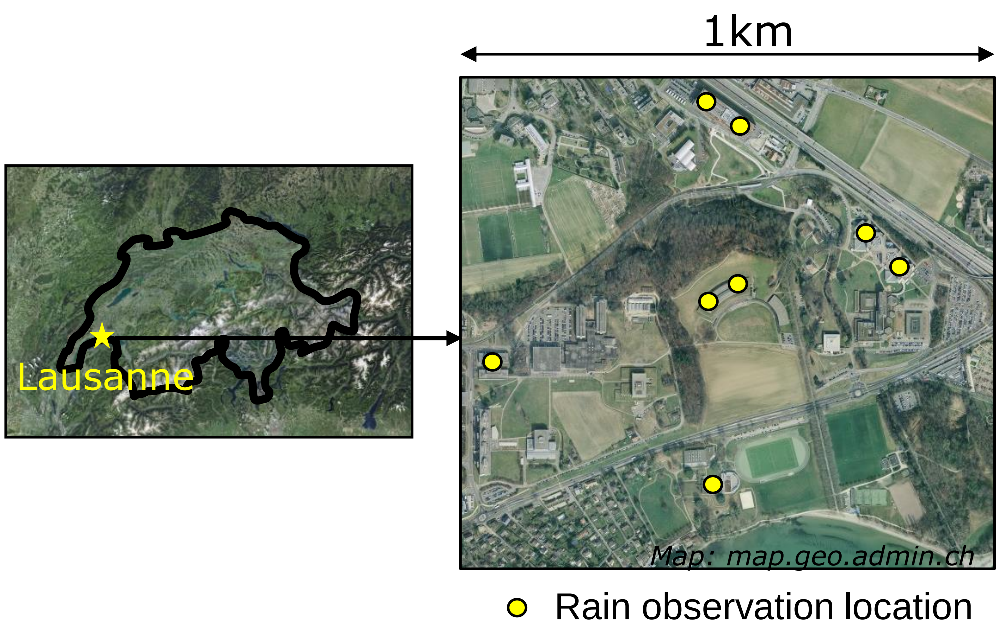


Application

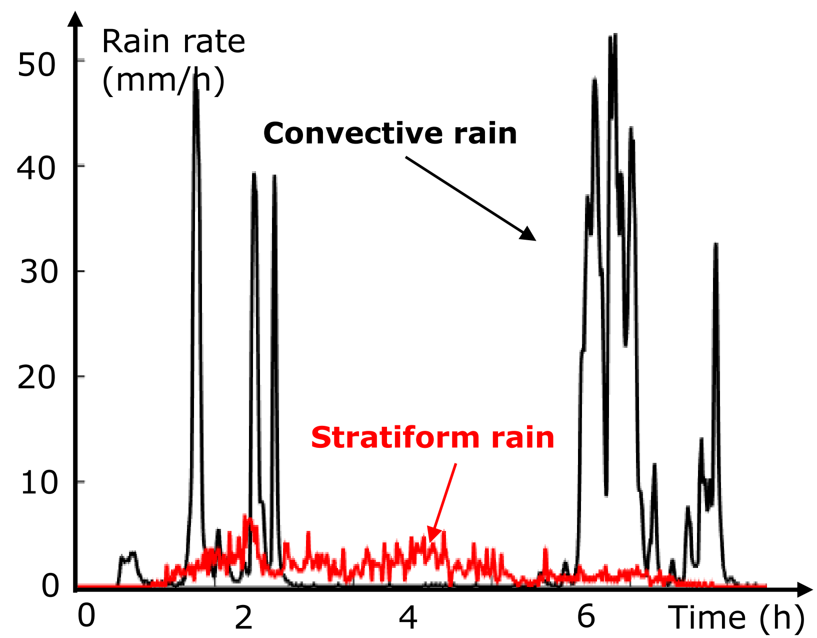
Experimental setup

- The proposed model is applied to two rain events in order to:
  - Assess the space-time structure of the rain events (parameter inference step).
  - Generate synthetic rain fields conditioned to observations (simulation step).

- Experimental setup:



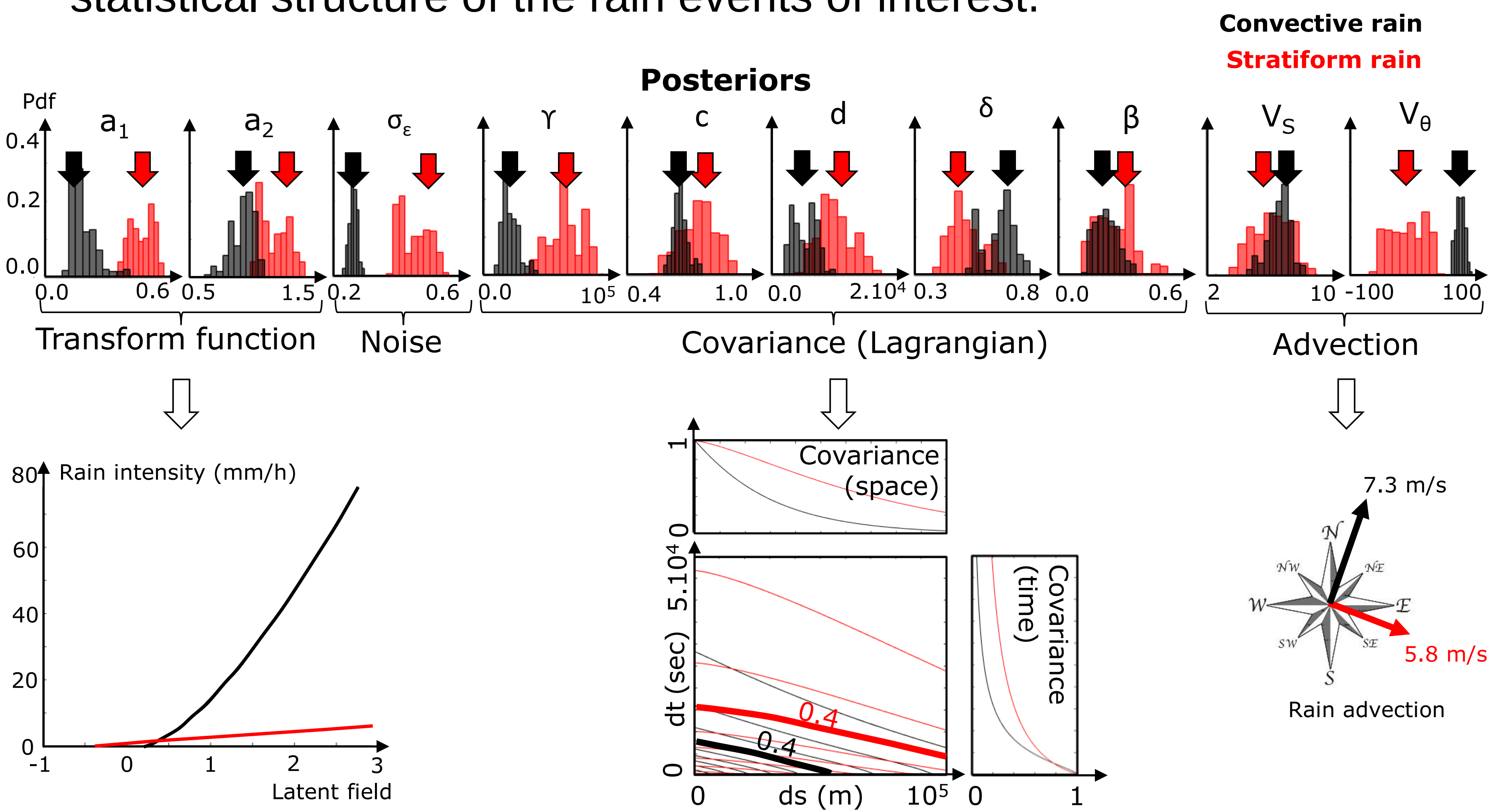
- Rain events of interest:



Application

Parameter inference

- Model parameters are inferred to assess the space – time – intensity statistical structure of the rain events of interest.

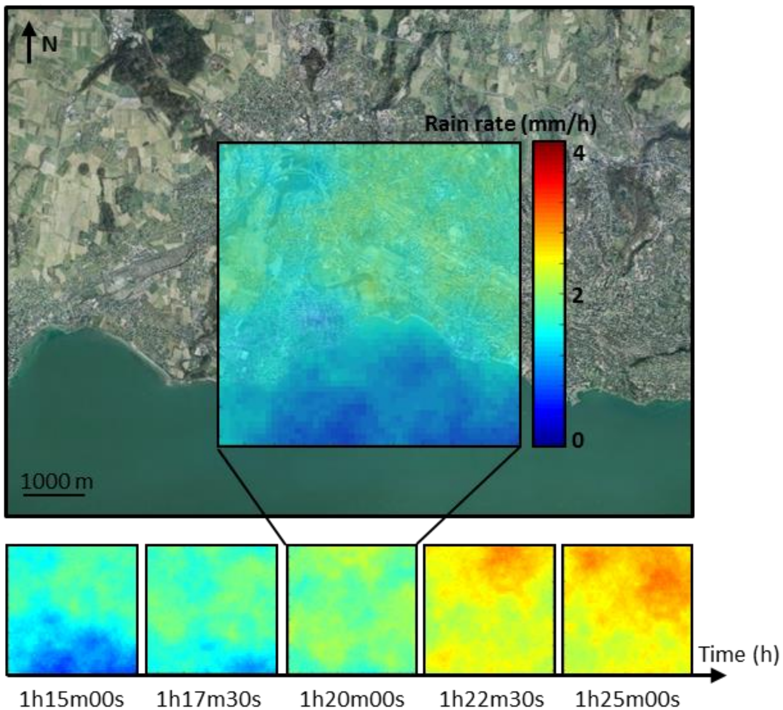


Application

Simulation of synthetic rain fields

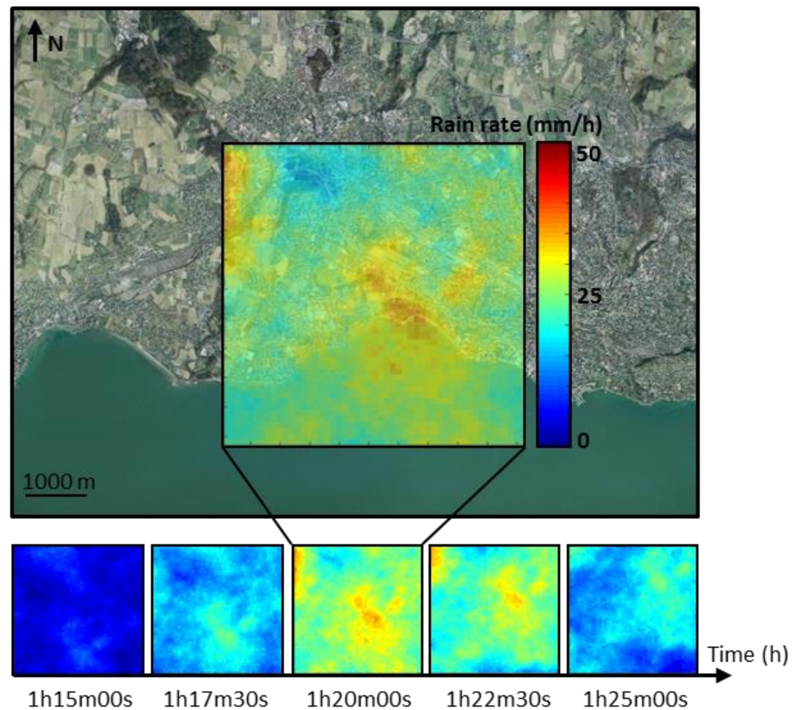
- The inferred model parameters lead to different simulated rain fields.

Stratiform rain



- Gentle rain, low intermittency.
- Low space-time variability.
- Eastward advection.

Convective rain

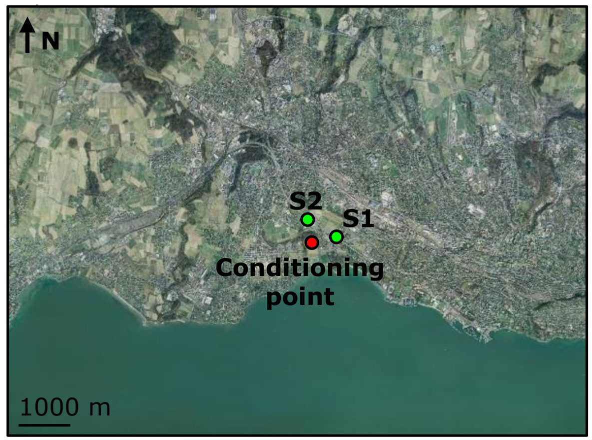


- Heavy rain, intermittent.
- High space-time variability.
- Northward advection.

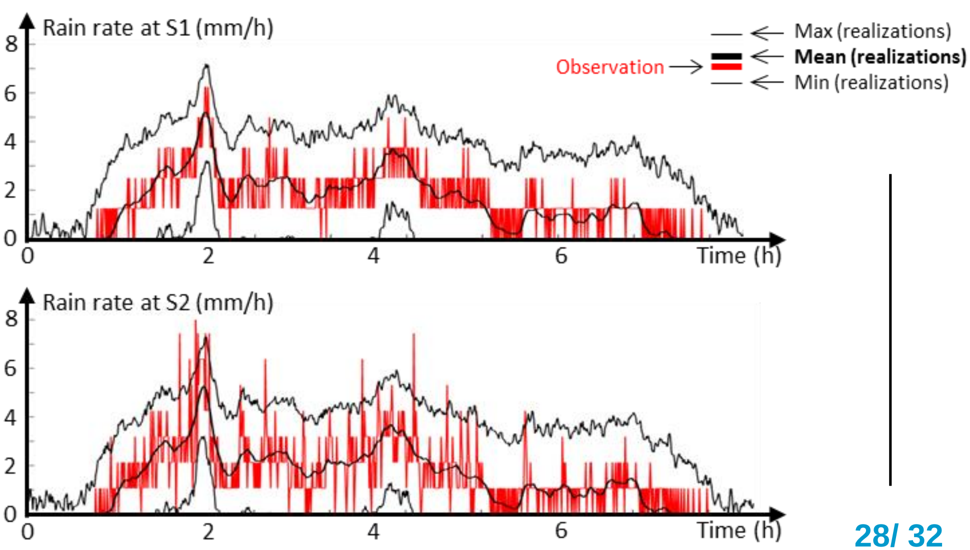
Application

Rain prediction

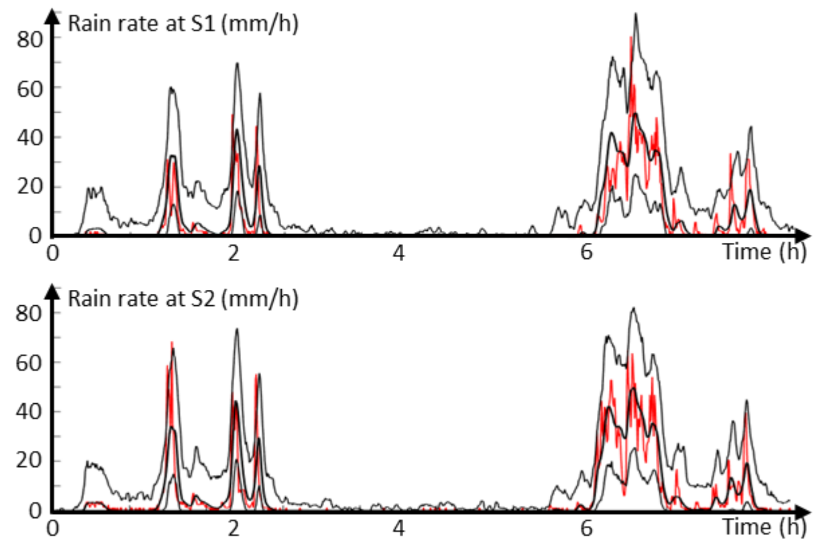
- An ensemble of 100 realizations can be used to predict rainfall at ungauged locations and to assess the prediction uncertainty.



Stratiform rain



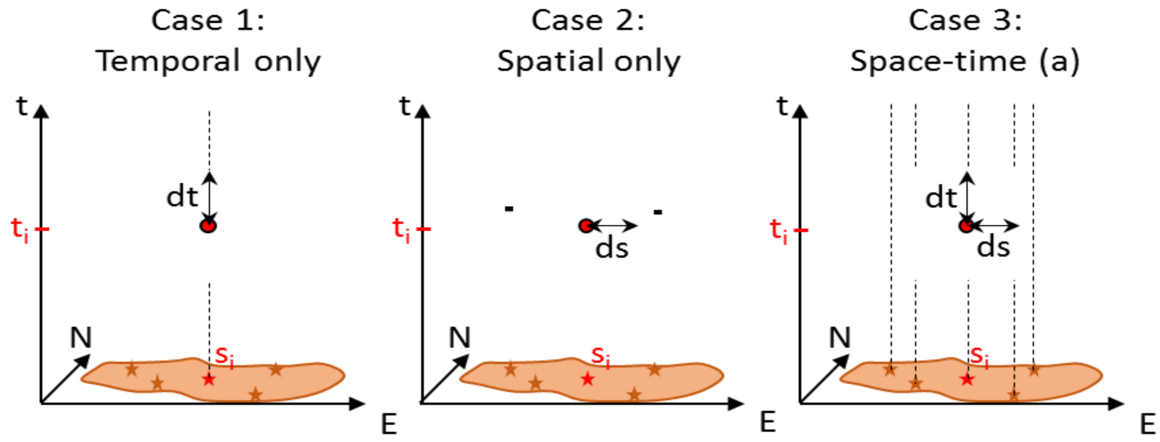
Convective rain



Application

Cross-validation

- Cross-validation to assess the reliability of rainfall prediction:



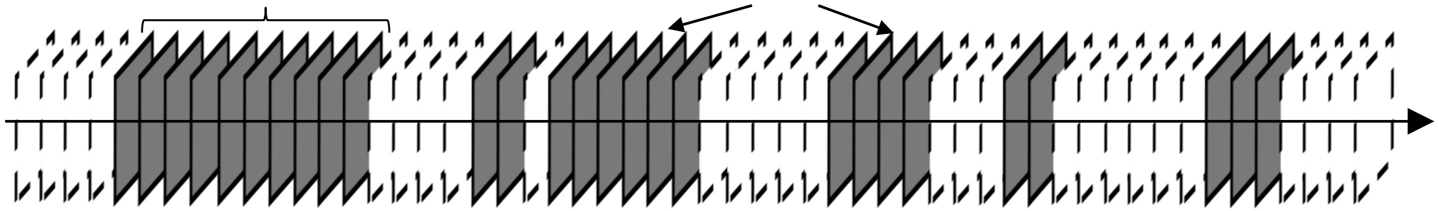
Stratiform rain	C1	C2	C3
dt = 30s ds = 50m	MAE = 0.70 Bias = -0.01	MAE = 0.69 Bias = -0.001	MAE = 0.71 Bias = -0.06
dt = 300s ds = 500m	MAE = 0.74 Bias = -0.01	MAE = 0.70 Bias = -0.002	MAE = 0.75 Bias = -0.006

Convective rain	C1	C2	C3
dt = 30s ds = 50m	MAE = 1.79 Bias = -0.02	MAE = 2.26 Bias = 0.30	MAE = 2.03 Bias = 0.29
dt = 300s ds = 500m	MAE = 3.83 Bias = 0.16	MAE = 1.99 Bias = 0.20	MAE = 2.28 Bias = 0.35

MAE: Mean Absolute Error

- Almost unbiased prediction.
- Low prediction errors.
- Positive contribution of the complex space-time covariance function.

Model summary



Model rainfall as a space-time random field

- Marginal distribution: Latent field + truncation & transform
- Space-time dependencies: Lagrangian + Space-time covariance

+

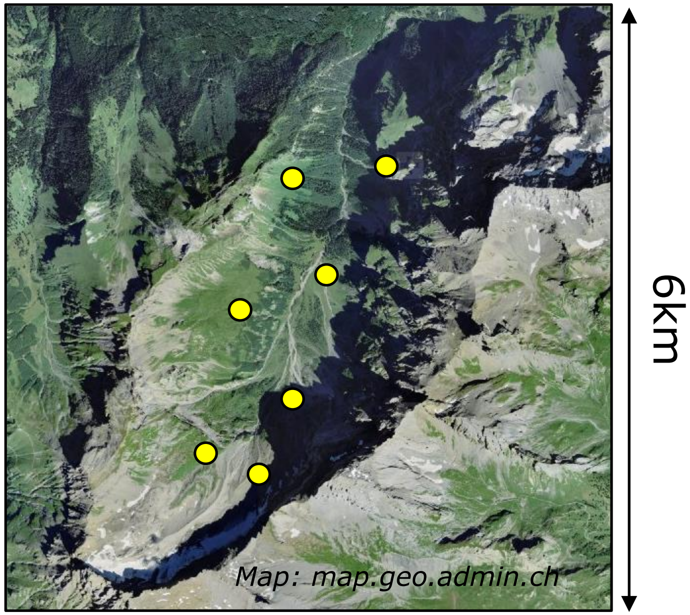
Parameter inference → Bayesian inference: Metropolis Hastings + Blockwise likelihood

+

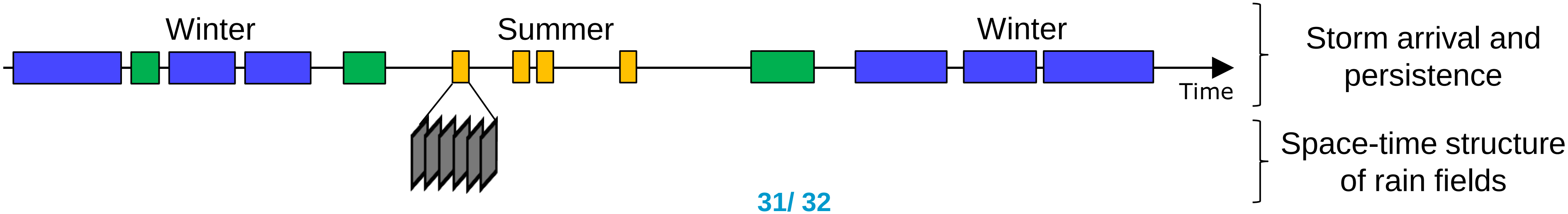
Simulation of synthetic rain fields → Geostatistical simulation: Multigrid simulation + back-transform

Perspectives

- Apply the proposed framework to a larger network:
  - 6km x 6km (Vallon de Nant, Swiss Alps).
  - Altitude: 1300 – 2200m => Orographic effects?
  - Application to mountain hydrology.



- Model storm arrival and persistence processes:
  - Account for seasonality in rainfall structure (rain types).
  - Application: stochastic rainfall generation for the whole year.



Conclusion

- Drop counting rain gauges allow to monitor local rain fields:
  - High resolution (0.01mm of rain) => high temporal resolution (30 sec).
  - Low cost, easy to set up => dense networks.

- A local scale stochastic rainfall model has been proposed to handle the features of rainfall arising from HR rain gauge measurements:
  - Rain intermittency and skewed distribution of positive rain rates.
  - Rain advection.
  - Temporal morphing of rain patterns.

- This model can be used to:
  - Investigate space-time dependencies within local rain fields.
  - Estimate HR rain fields over small catchments (Mountain / urban hydrology).

Thank you for your attention

- Drop counting rain gauges allow to monitor local rain fields.
- A Stochastic rainfall model has been proposed to handle these data.
- Applications: - Investigate space-time dependencies within rain fields.

  - Estimate HR rain fields over small catchments.

Picture: Anthony Michelon