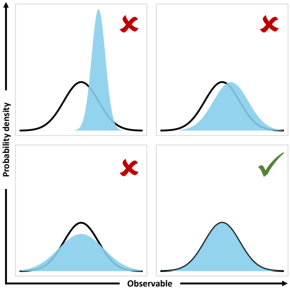


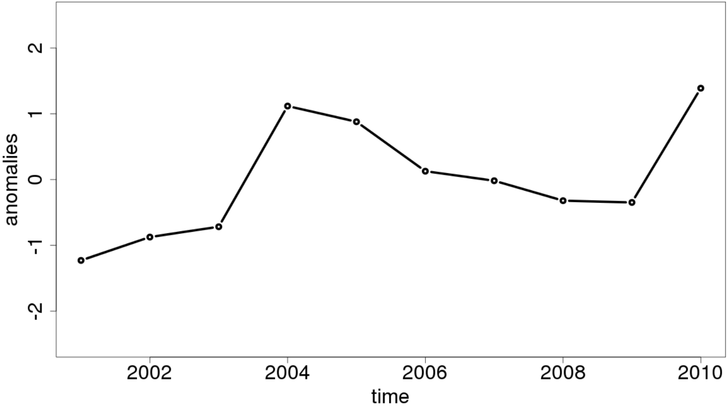
Calibration of decadal ensemble predictions

A. Pasternack, H. W. Rust, U. Ulbrich, M. A. Liniger, J. Bhend
Freie Universität Berlin

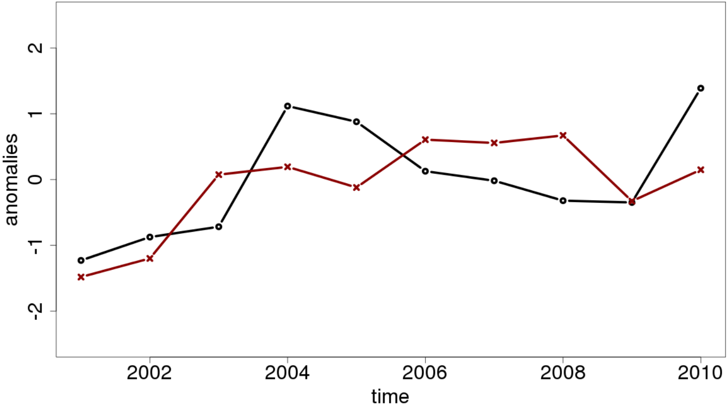
Berlin, October, 5th, 2016

- Probabilistic forecasts
- What is a good forecast
- Re-calibrating an example forecast
- Tailor re-calibration methods to decadal predictions
- Apply re-calibration methods to decadal predictions
  - Validation

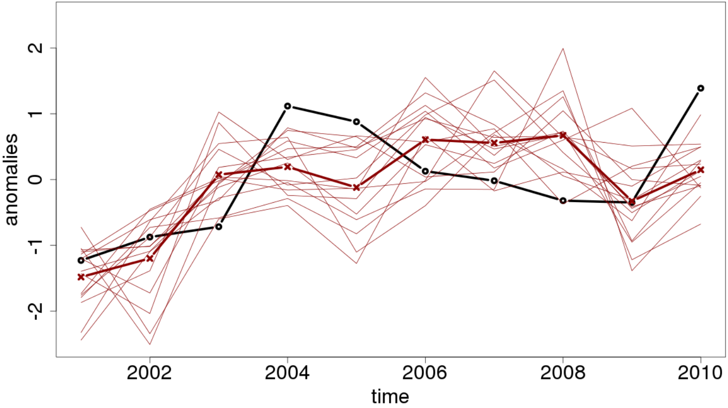
Probabilistic forecast



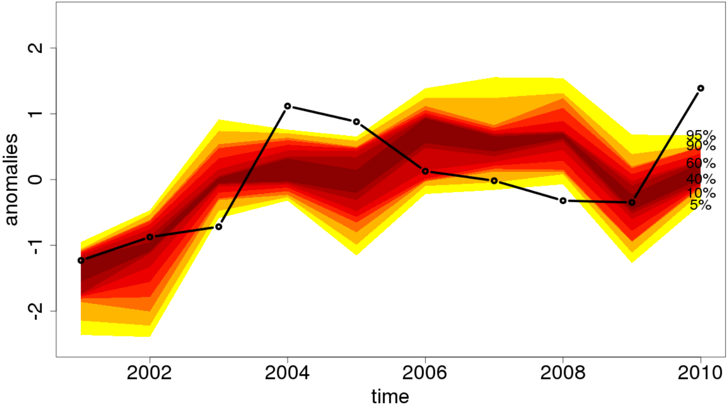
Probabilistic forecast



Probabilistic forecast



Probabilistic forecast



What is a good probabilistic forecast?

„... an important goal is to maximize <u>sharpness</u> without sacrificing <u>calibration</u>.“ (Wilks, 2011; Gneiting, 2007; Murphy and Winkler, 1987)

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Calibration or reliability

Probabilistic forecasts „mean what they say“, e.g. for days with a forecast of 30% chance of rain, we expect a relative frequency of 30% rainy days.

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„Ensemble members are reliable if the MSE between the ensemble mean and observations is identical to the time mean intra-ensemble variance." (Palmer et al., 2006)

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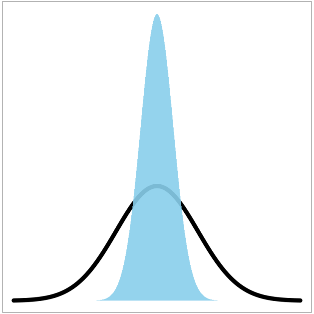

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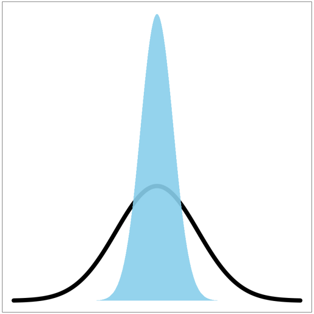
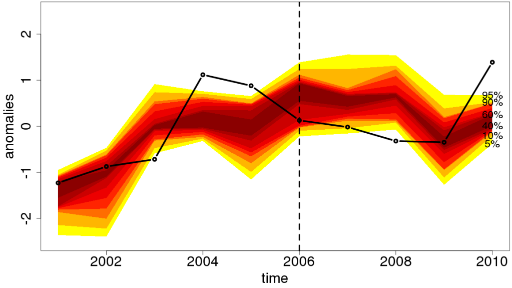
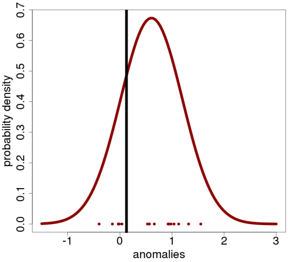
⟶ **adjust ensemble spread**



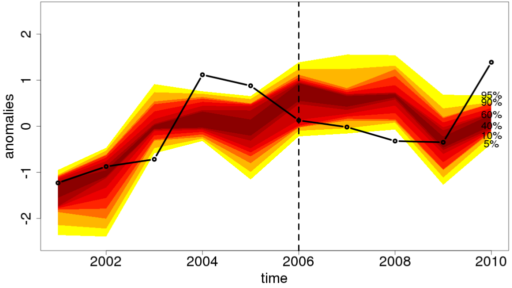
Re-calibrating an example forecast

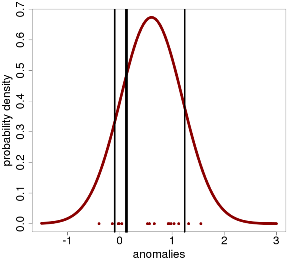
Probabilistic forecast





Probabilistic forecast





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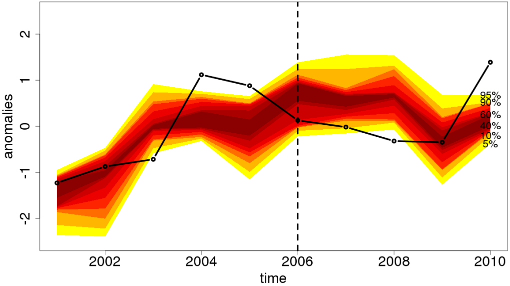

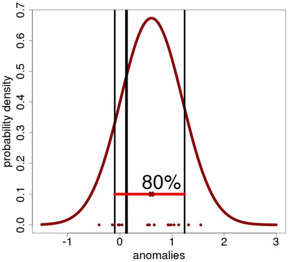

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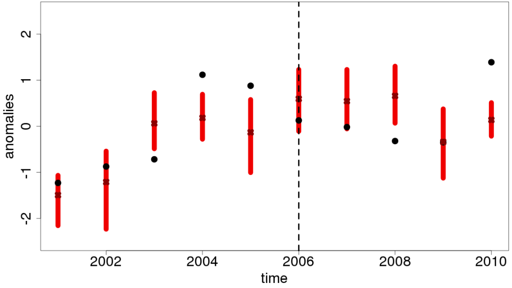
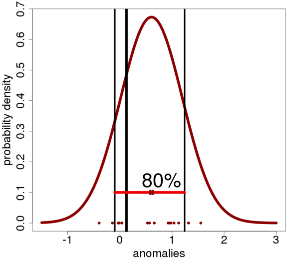


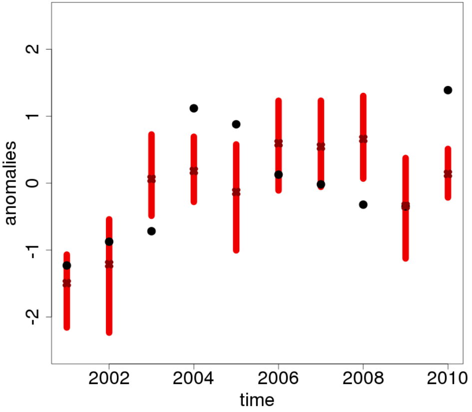


Reliable forecast:

- 80% of ens. spread should include 80% of observations
- only 50% are covered

What is wrong?

Re-calibrating an example forecast

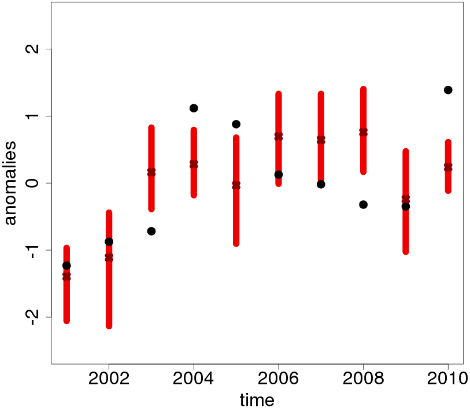


1. forecast is biased (uncond.)

Ensemble prediction:

$f_i(t) = \mu(t) + \epsilon_i(t) \qquad i = 1 \ldots M$

Re-calibrating an example forecast



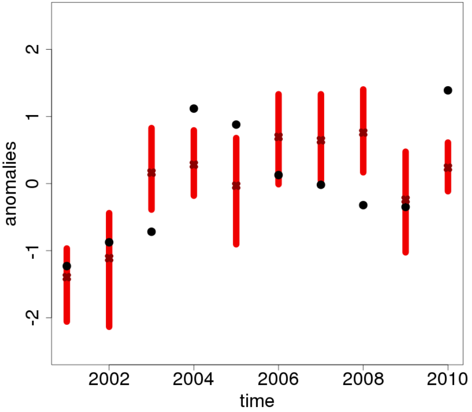
1. forecast is biased (uncond.) (shift ensemble mean)

2. forecast is conditionally biased

Ensemble prediction:

$f_i(t) = \alpha + \mu(t) + \epsilon_i(t) \qquad i = 1 \ldots M$

Re-calibrating an example forecast



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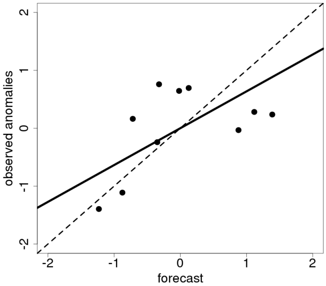

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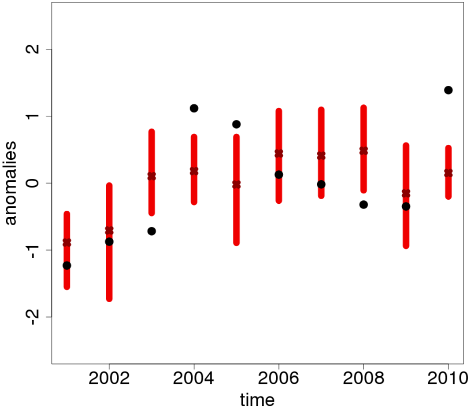

1. forecast is biased (uncond.)
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2. forecast is conditionally biased
(scale ensemble mean)

3. forecast is not reliable

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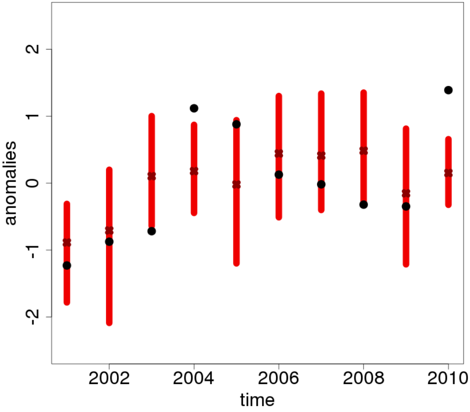
Re-calibrating an example forecast



1. forecast is biased (uncond.)
(shift ensemble mean)

2. forecast is conditionally biased
(scale ensemble mean)

3. forecast is not reliable
(scale ensemble spread)

4. re-calibrated forecast

Re-calibrated ensemble:

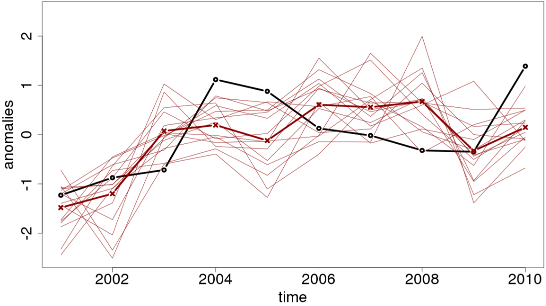
$$f_i^{Cal}(t) = \alpha + \beta\mu(t) + \gamma\epsilon_i(t) \qquad i = 1 \ldots M$$

Re-calibrating an example forecast

State of the art:

Re-calibration is used in

- weather prediction
- seasonal prediction



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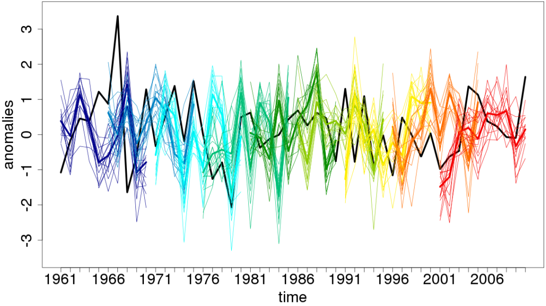
Tasks for CALIBRATION:

Tailor re-calibration methods to decadal predictions



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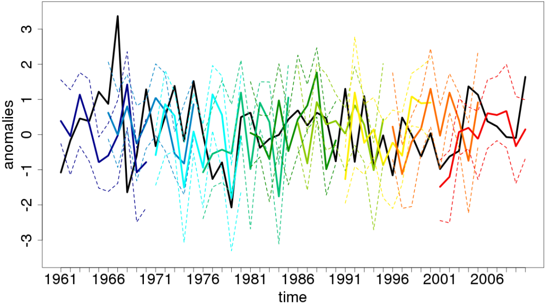

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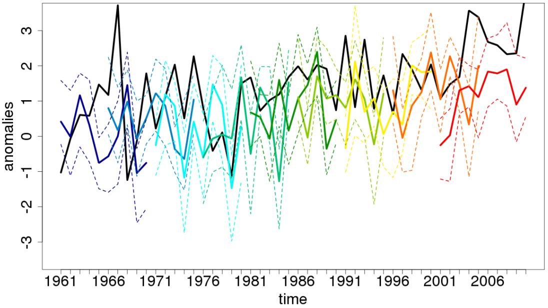

Tasks for CALIBRATION:

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Re-calibrating an example forecast

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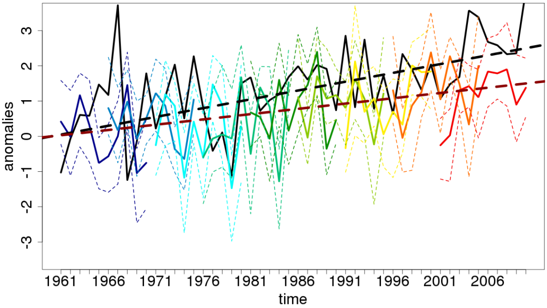

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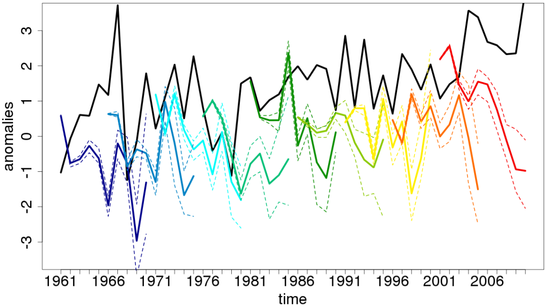

Tasks for CALIBRATION:

Tailor re-calibration methods to decadal predictions

- limited number of hindcasts
- climate trend
- dependence on lead years (drift)

Ensemble prediction

$$f_i(t,\tau) = \mu(t,\tau) + \epsilon_i(t,\tau)$$

$i = 1...M$ ensemble member, $t =$ start year, $\tau =$ lead year

with

$$\mu(t,\tau) = E(f_i(t,\tau))$$

Re-calibrated ensemble

$$f_i^{Cal}(t,\tau) = \alpha(t,\tau) + \beta(t,\tau)\mu(t,\tau) + \gamma(t,\tau)\epsilon_i(t,\tau)$$

find $\alpha(t,\tau)$, $\beta(t,\tau)$ and $\gamma(t,\tau)$ such that the ensemble is perfectly calibrated with maximum sharpness

1) α: bias and drift, 2) β: conditional bias, 3) γ: ensemble spread

General idea

Minimize continuous ranked probability score (crps) between model f^{Cal} and observation O (*Gneiting et al. 2005*):

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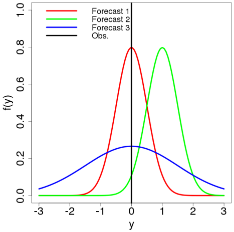

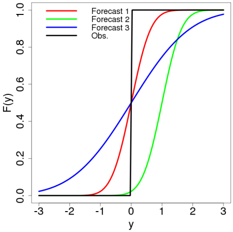

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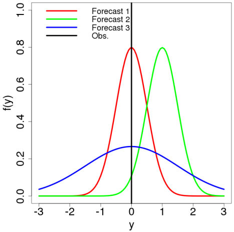

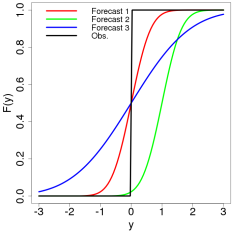

$crps_1 = 0.12$, $crps_2 = 0.73$, $crps_3 = 0.35$

crps measures:

- reliability
- sharpness

General idea (2)

If the forecast distribution is Gaussian:

$$\longrightarrow f_i^{Cal}(t,\tau) \sim \mathcal{N}(\alpha(t,\tau) + \beta(t,\tau)\mu(t,\tau), \gamma(t,\tau)^2\sigma^2(t,\tau))$$

General idea (2)

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...the crps simplifies to:

$$crps(\mathcal{N}(\mu,\sigma^2),o) = \sigma\{\frac{o-\mu}{\sigma}[2\Phi(\frac{o-\mu}{\sigma}) - 1] + 2\varphi(\frac{o-\mu}{\sigma}) - \frac{1}{\sqrt{\pi}}\}$$

$\mu =$ens. mean, $\sigma =$ens. std., $o =$observation, $\Phi,\varphi =$ CDF and PDF of stand. norm. distr.

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$\mu =$ens. mean, $\sigma =$ens. std., $o =$observation, $\Phi,\varphi =$ CDF and PDF of stand. norm. distr.

The average score over all k pairs of forecasts and observations is:

$$\Gamma(\mathcal{N}(\alpha + \beta\mu, \gamma^2\sigma^2), o) = \frac{1}{k}\sum_{j=1}^{k}\sqrt{\gamma^2\sigma_j^2}\{Z_j[2\Phi(Z_j) - 1] + 2\varphi(Z_j) - \frac{1}{\sqrt{\pi}}\},$$

$$Z_j = \frac{O_j - (\alpha + \beta\mu_j)}{\sqrt{\gamma^2\sigma_j^2}}$$

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$$Z_j = \frac{O_j - (\alpha + \beta\mu_j)}{\sqrt{\gamma^2\sigma_j^2}}$$

$\alpha = \alpha(t,\tau) = (a_0 + a_1 t) + (a_2 + a_3 t)\tau + (a_4 + a_5 t)\tau^2 + (a_6 + a_7 t)\tau^3 + \ldots$
$\beta = \beta(t,\tau) = (b_0 + b_1 t) + (b_2 + b_3 t)\tau + (b_4 + b_5 t)\tau^2 + (b_6 + b_7 t)\tau^3 + \ldots$
$\gamma = \gamma(t,\tau) = (c_0 + c_1 t) + (c_2 + c_3 t)\tau + (c_4 + c_5 t)\tau^2 + (c_6 + c_7 t)\tau^3 + \ldots$

$\longrightarrow$ find an $a_0, b_0, c_0, \ldots, a_7, b_7, c_7$ that minimize Γ

Example: Surface Temperature

Data:

- Surface temperature over the North Atlantic region
- Model: MPI-ESM-LR, Prototype (GECCO2)
- 15 ensemble members
- Initialisation years: 1961-2000
- Annual mean
- Reference: NCEP 20CR

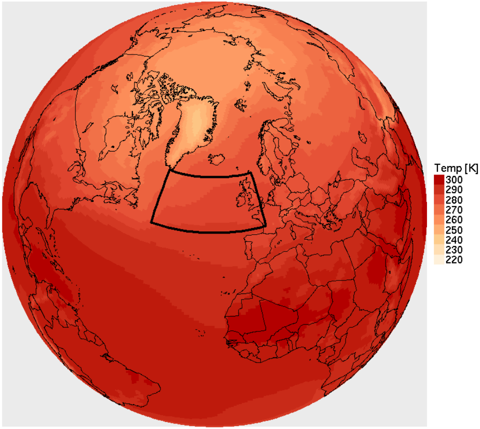
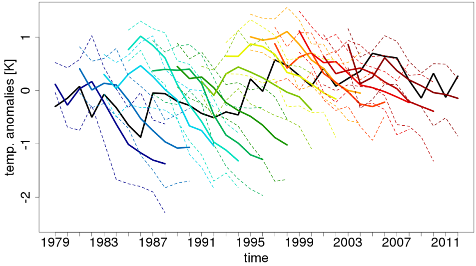


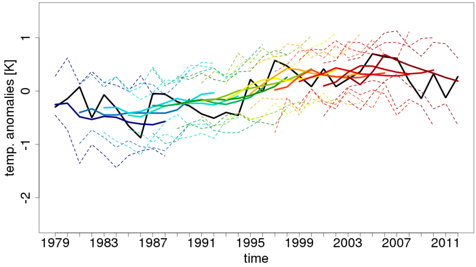
Figure: ST time mean for NCEP 20CR

Apply re-calibration method to decadal prediction

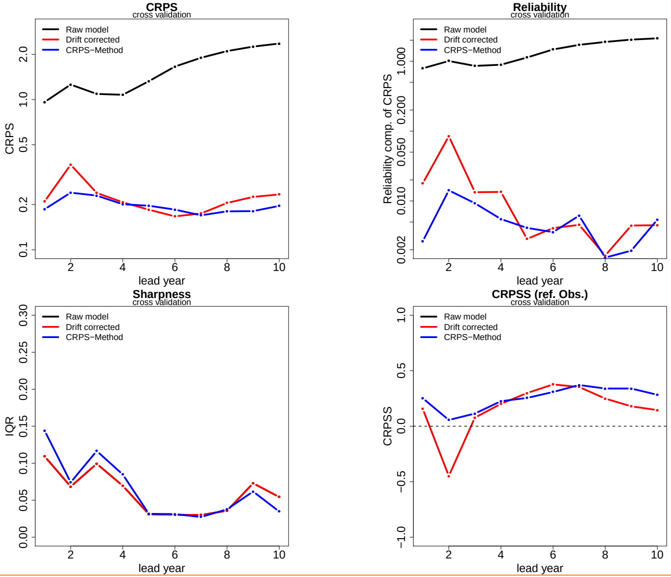
Before:



After:



Validation


CRPS
cross validation
Raw model
Drift corrected
CRPS−Method
CRPS
0.1
0.2
0.5
1.0
2.0
2
4
6
8
10
lead year
Reliability
cross validation
Raw model
Drift corrected
CRPS−Method
Reliability comp. of CRPS
0.002
0.010
0.050
0.200
1.000
lead year
Sharpness
cross validation
Raw model
Drift corrected
CRPS−Method
IQR
0.00
0.05
0.10
0.15
0.20
0.25
0.30
lead year
CRPSS (ref. Obs.)
cross validation
Raw model
Drift corrected
CRPS−Method
CRPSS
−1.0
−0.5
0.0
0.5
1.0
lead year

What is a good probabilistic forecast?

„... an important goal is to maximize sharpness without sacrificing reliability.“ (Wilks, 2011; Gneiting, 2007; Murphy and Winkler, 1987)

- CRPS minimization method by *Gneiting et al. 2005* addresses to reliability and sharpness for seasonal prediction.
- The developed extension to decadal predictions also includes a lead time dependent drift correction.

Validation

- CRPS method is mostly superior to drift correction and climatology w.r.t. predictive skill.
- Sharpness will be decreased to obtain good reliability.

Thank you for your attention!

References

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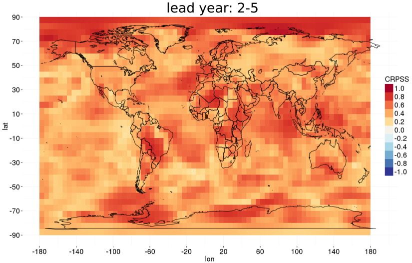
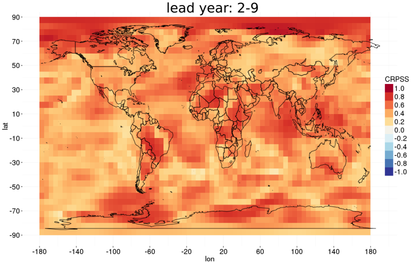
- Gneiting, T., and A. E. Raftery, 2007: Strictly proper scoring rules, prediction, and estimation. Journal of the American Statistical Association, 102 (477), 359–378.
- Weigel, A., M. A. Liniger, and C. Appenzeller., 2009: Seasonal ensemble forecasts: Are recalibrated single models better than multimodels? Mon. Weather Rev., 137(4), 1460–1479.
- Wilks,D.S.,2011:Statistical Methods in the Atmospheric Sciences.Academic Press.

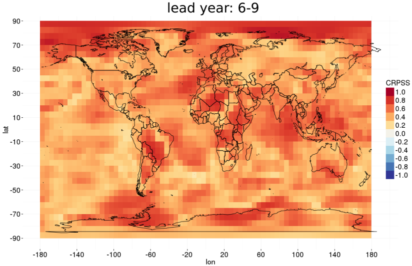
Appendix

Spatial Analyses (1)

CRPSS: Re-calibrated vs. Climatology (cross validation)







Spatial Analyses (2)

CRPSS: Re-calibrated vs. Drift Corrected (cross validation)