$E[\hat{\Theta}]-\Theta_0$ | BERLIN **2016**

How to define a bias for climate models?

Some thoughts about the concept of bias as used for weather and climate models

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Bias

for weather & climate models?

Statistics

$$b = E\left[\hat{\theta}\right] - \theta_0$$

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Weather & climate (forecast verification)

$$\mathrm{ME} = \frac{1}{N}\sum_{i=1}^{N}(F_i - O_i)$$

(WWRP/WGNE, 2009)

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Bias in statistics

Transferring the concept to weather & climate models

Expected error

Bias correction

Estimating the expectation

Bias in statistics

Bias in statistics

$$b = E\left[\hat{\theta}\right] - \theta_0$$

- $E[.]$ expectation,
- $\hat{\theta}$ estimator for θ, or statistic,
- θ_0 true value

Example: arithmetic mean

An estimator for the expectation of RV X_i

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

Bias of the arithmetic mean

$$b = E[\hat{\mu}] - \mu$$

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Suppose: random variables X_i with $E[X_i] = \mu$

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$$= \mu$$

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$$b = E[\hat{\mu}] - \mu = \mu - \mu = 0$$

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Note:

for a less trivial examples, consider the bias of $s^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \hat{\mu})^2$

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- expectation $E[.]$ of a random variable
- estimator $\hat{\mu}$ (random variable)
- true value μ

Transferring concepts to weather & climate models

$$b = E[\hat{\theta}] - \theta$$

1. What is the true value?

2. What to do with the expectation?

3. What is the estimator in case of NWPs/GCMs?

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- station based measurements,
- remote sensing, e.g. ground-based radar,
- remote sensing, e.g. space-born radar,
- reanalyses or
- interpolated station data?

Die Wahrheit ist verborgen.

Wir müssen sie zu schätzen wissen!

The truth is unkown.

Appreciate it’s estimation!

The measuring process as a random variable

Imagine reading an analog/digital thermometer. . .

Types of error:

The measuring process as a random variable

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Types of error:

systematic construction issues

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- **systematic** construction issues
- **random** reading value depends on viewing angle/size of person reading (analog), rounding to finite precision (digital)

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Imagine reading an analog/digital thermometer. . .

Types of error:

systematic construction issues

random reading value depends on viewing angle/size of person reading (analog), rounding to finite precision (digital)

For a quantity θ with unknown true value, conceive the measuring process as a statistic T_o.

Definition: bias of measurement process

$$b_o = E\left[T_o\right] - \theta$$

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- $E\left[T_o\right]$ cannot be computed unless having a PDF, needs to be estimated
- the true value θ remains unknown

What to do with the expectation?

What to do with the expectation?

Instead of computing the arithmetic mean, conceive the quantity under consideration as a random variable with an expectation.

What is the estimator in the case of NWPs/GCMs?

Model output as estimator

Model output for the quantity under consideration, e.g.

- daily mean 2-m temperature,
- monthly precipitation sums,
- daily temperature range
- annual variability of daily precip sums
- 0.9-quantiles of daily precip sums
- ...

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Arguments

- characteristics of a RV: imagine some true value and a variation around, e.g. due to dependence on initial conditions
- time scale dependent:

climate models	expectation is a climatology, result varies with initialization, use any admissable state under given climate conditions
weather model	expectation is weather, result varies with initialization, use likely states given the observational uncertainties.

The model output as a random variable

Imagine output of a NWP/GCM. . .

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For a quantity θ with unknown true value, conceive the model output as a statistic T_m.

Definition: bias of model output

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$$b = E[\hat{\theta}] - \theta$$

1. What is the true value?
   Unknown! Conceive measurement process as RV and estimate

2. What to do with the expectation?
   Conceive quantity as RV and estimate

3. What is the estimator in case of NWPs/GCMs?
   Conceive NWPs/GCM as the estimator

Definition: bias of observation

$$b_o = E[T_o] - \theta$$

Definition: bias of model output

$$b_m = E[T_m] - \theta$$

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$$= \frac{1}{N}\sum_{i=1}^{N}F_i - \frac{1}{N}\sum_{i=1}^{N}O_i$$

Expected error

$$\mathrm{EE} = E[T_m] - E[T_o]$$

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insert $0 = \theta - \theta$

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Difference in biases

$$\begin{aligned}\mathrm{EE} &= E\left[T_m\right] - \theta + \theta - E\left[T_o\right] \\ &= \left(E\left[T_m\right] - \theta\right) - \left(E\left[T_o\right] - \theta\right) \\ &= b_m - b_o\end{aligned}$$

Expected error

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Mean error

$$\mathrm{ME} = \frac{1}{N}\sum_{i=1}^{N}(F_i - O_i)$$

is an estimate of the difference between model bias and observation bias.

Bias correction

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Subtracting the estimated expected error from a quantity means to replace it with the bias of the observation

$$T_m - EE = T_m - b_m + b_o$$

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Subtracting the estimated expected error from a quantity means to replace it with the bias of the observation

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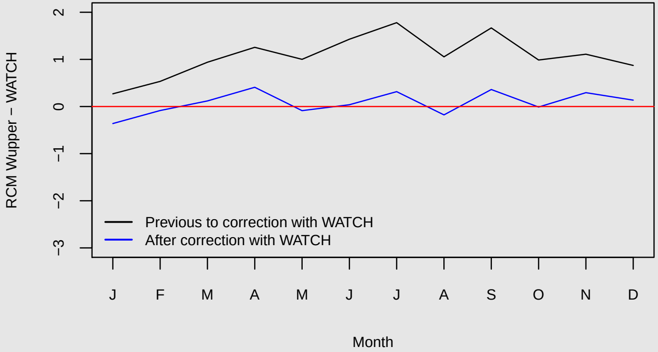
Thus the bias of the „bias corrected“ quantity is

$$\begin{aligned} b &= E\left[T_m - EE\right] - \theta = E\left[T_m\right] - \theta - b_m + b_o \\ &= b_m - b_m + b_o = b_o \end{aligned}$$

Example: RCM for the Wupper catchment

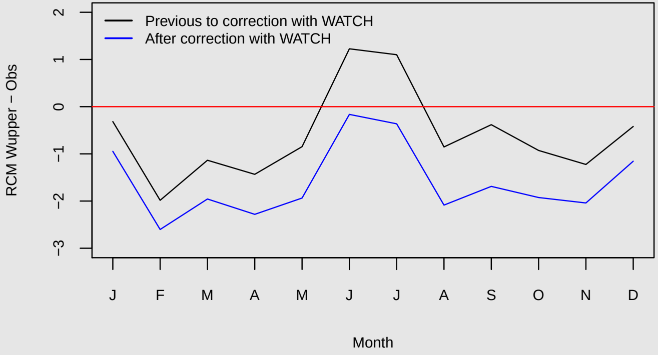
Monthly mean precip, BC with WATCH and verified against WATCH



Example: RCM for the Wupper catchment

Monthly mean precip, BC with WATCH and

verified against Obs



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- bias correction means removing model bias and replacing it with observation bias
- using expectation instead of arithmetic mean allows for other estimation strategies

Estimating the expectation

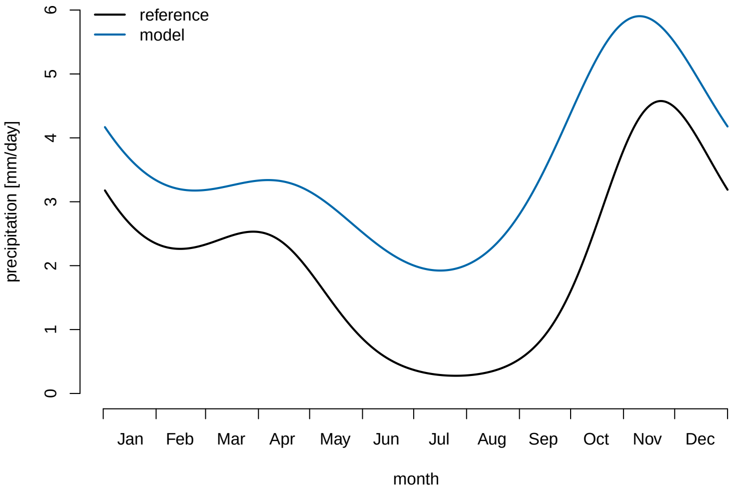
Example: seasonality in daily precipitation

Using generalized linear models

$$RR \sim \text{Gamma}(\theta)$$
$$\theta = E[Pr]$$
$$h(\theta) = \mu_0 + \sum_{k=1}^{K} \sin(\omega_k t) + \sum_{l=1}^{L} \cos(\omega_l t)$$

with $\omega_k = k\,2\pi/365.25$ and t the day of the year.

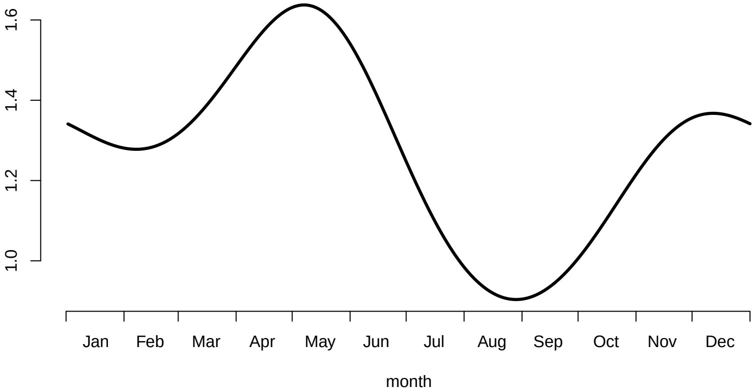
Example: seasonality with GLMs



Daily precipitation sums at a grid point in the Wupper catchment

Example: seasonality with GLMs

Estimate of the expected error



Ratio of daily precipitation sums at a grid point in the Wupper catchment

See poster by Madlen Fischer.

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- relationships between variables can be maintained (Petra, Alex C)

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Enjoy!