- Various Displacement Well Response Testing -
A Well Performance Testing Methodology to Identify Nonlinear Formation-Controlled Flow

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- Motivation – Why well testing?
- Status Quo – Classical linear slug test theories
- Observed anomalies
- A new fully nonlinear slug test model
- Applications of the new nonlinear slug test model
- Perspectives – Testing of low-permeability formations
- Summary
Why well testing?

- **Well testing can identify reservoir complexity** (e.g. reservoir boundaries)! Reservoir complexity usually is unknown in the early stage of an investigation but knowledge of reservoir complexity is needed to render numerical reservoir simulation meaningful.

- **Well testing directly provides „upscaled“ hydraulic parameters** (needed for numerical flow simulation).

- **Well testing allows to identify reservoir leakage** (important to determine reservoir/aquifer integrity).

- **Well testing provides well productivities/injectivities** (important to determine the economical feasibility of geothermal/hydrocarbon exploitation and subsurface storage projects).

- **This presentation shows:** Various displacement well response testing allows to identify nonlinear formation-controlled flow.
  - supports an identification of hydraulically conductive fractures
  - relevant to cap rock tightness characterization
  - relevant to fractured/karstified reservoir characterization
Status Quo - Classical linear slug test theories

\[ H(t) \]

land surface

static level

\[ r_c \]

\[ r_p \]

\[ r_s \]

\[ L_p \]

\[ z_0 \]

\[ D \]

\[ M = B \]
Cooper-Bredhoefft-Papadopulos (CBP) Model (1967):

\[
\frac{H(t)}{H_0} = \frac{8r_s^2 S}{\pi^2 r_c^2} \cdot \int_0^\infty \frac{1}{u \cdot \Delta u} \exp \left\{ - \frac{Ttu^2}{r_s^2 S} \right\} du
\]

The right-hand side is independent of \(H_0\)!
Classical normalized head responses are convex or linear collapsing onto a unique curve for different $H_0$: 
The Problem: We have observed concave normalized head responses not collapsing onto a unique curve:
A new fully nonlinear slug test model

\textbf{Fully nonlinear slug test model for finite diameter wells (Zenner, 2008, 2009):}

\begin{align*}
- \left( H + z_0 + \left[ \frac{r_c^2}{r_p^2} - 1 \right] L_p + \left[ \frac{3}{8} B + D \right] \frac{r_c^2}{r_s^2} \right) \frac{d^2 H}{dt^2} - gH \\
+ \frac{1}{2} \left[ \left( \frac{r_c^2}{2r_s^2 B} \right)^2 - 1 + \xi_{\text{loss}} \right] - \left( f_p \frac{L_p r_c^4}{2r_p^5} + f_s \frac{D r_c^4}{2r_s^5} + f_c \frac{z_0 - L_p + H}{2r_c} \right) \text{sign} \left( \frac{dH}{dt} \right) \left( \frac{dH}{dt} \right)^2 \\
- g \pi r_c^2 \left( B_2 + C \left( \pi r_c^2 \right)^{p-1} \right) \frac{dH}{dt} \left( \frac{dH}{dt} \right) - \frac{\left( \sqrt{r_c^2} \right)^2}{2 \pi^2 T} \int_0^t \int_0^\infty \frac{1 - e^{-\tau}}{S r_D^2} x^3 \left[ J_1^2(2x) + Y_1^2(2x) \right] dx d\tau = 0
\end{align*}
Applications of the new nonlinear slug test model

Example No. 1:
Slug test analysis at well B-7004 (Berlin, Tempelhof Airport)

Aquifer thickness

10 m thick clay sea

Filter gravel pack 1-2 mm

Marl

Medium sand, coarse sand

Medium sand, fine sand

Clay, sandy

Fine sand

Sandy clay

Marl

Filter gravel pack 1-2 mm

Aquifer thickness
Applications of the new nonlinear slug test model

Test design to investigate packer-related nonlinear head loss components:

<table>
<thead>
<tr>
<th>Slug Test Identifier</th>
<th>$r_p$ (m)</th>
<th>$L_p$ (m)</th>
<th>$H_0$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B7004/1</td>
<td>0.025</td>
<td>1.774</td>
<td>3.96</td>
</tr>
<tr>
<td>B7004/2</td>
<td>0.025</td>
<td>1.774</td>
<td>1.35</td>
</tr>
<tr>
<td>B7004/3</td>
<td>0.025</td>
<td>9.774</td>
<td>4.15</td>
</tr>
<tr>
<td>B7004/4</td>
<td>0.025</td>
<td>9.774</td>
<td>1.35</td>
</tr>
<tr>
<td>B7004/5</td>
<td>0.014</td>
<td>1.774</td>
<td>4.12</td>
</tr>
<tr>
<td>B7004/6</td>
<td>0.014</td>
<td>1.774</td>
<td>1.33</td>
</tr>
<tr>
<td>B7004/7</td>
<td>0.0055</td>
<td>1.774</td>
<td>4.09</td>
</tr>
<tr>
<td>B7004/8</td>
<td>0.0055</td>
<td>1.774</td>
<td>1.33</td>
</tr>
<tr>
<td>B7004/11</td>
<td>0.014</td>
<td>9.774</td>
<td>4.21</td>
</tr>
<tr>
<td>B7004/12</td>
<td>0.014</td>
<td>9.774</td>
<td>1.36</td>
</tr>
</tbody>
</table>
Applications of the new nonlinear slug test model

Measured vs. simulated responses at well B-7004:

- Data of test B7004/1
- Data of test B7004/2
- Data of test B7004/3
- Data of test B7004/4
- Data of test B7004/5
- Data of test B7004/6
- Data of test B7004/7
- Data of test B7004/8
- Data of test B7004/11
- Data of test B7004/12
- Simulations based on the nonlinear slug test model
Inferences from Example No. 1:

- Nonlinear tubing-controlled flow causes concavity and implies a shift of normalized head responses toward larger times.

- Colebrook and Borda Carnot-type head loss formulas from steady state pipe hydraulics are sufficiently accurate at modeling tubing-controlled transient flow inside the well.
Applications of the new nonlinear slug test model

Example No. 2:
Slug test analysis at well Münstereifelbohrung B2 (Eifel-Area)

Land Surface: 417.35 m NN

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Limestone description</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.00</td>
<td>brown</td>
</tr>
<tr>
<td>15.00</td>
<td>pale grey</td>
</tr>
<tr>
<td>23.00</td>
<td>reddish brown</td>
</tr>
<tr>
<td>29.00</td>
<td>pale brown</td>
</tr>
<tr>
<td>43.00</td>
<td>pale grey</td>
</tr>
<tr>
<td>53.00</td>
<td>dark-brown</td>
</tr>
<tr>
<td>61.00</td>
<td>pale brown</td>
</tr>
<tr>
<td>72.00</td>
<td>auburn</td>
</tr>
<tr>
<td>89.00</td>
<td>pale grey</td>
</tr>
<tr>
<td>101.00</td>
<td>hard, grey</td>
</tr>
</tbody>
</table>

Overburden

Clay seal
Drilling radius $r_d = 0.120$ m
Static water level on October 25, 2006 = 60.23 m
Casing radius $r_c = 0.0625$ m
Clay seal
Filter gravel 2 - 3 mm
Screen radius $r_s = 0.0625$ m
Applications of the new nonlinear slug test model

Test design to investigate the flow dynamics inside the fractured Devonian limestone formation:

<table>
<thead>
<tr>
<th>Slug Test Identifier</th>
<th>( r_p ) (m)</th>
<th>( L_p ) (m)</th>
<th>( C ) (s(^p/m^3p^{-1}))</th>
<th>( p ) (-)</th>
<th>( H_0 ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mue2/1</td>
<td>0.0625</td>
<td>0</td>
<td>23500</td>
<td>1.6</td>
<td>- 0.715</td>
</tr>
<tr>
<td>Mue2/5</td>
<td>0.0625</td>
<td>0</td>
<td>23500</td>
<td>1.6</td>
<td>- 0.475</td>
</tr>
<tr>
<td>Mue2/7</td>
<td>0.0625</td>
<td>0</td>
<td>23500</td>
<td>1.6</td>
<td>- 0.129</td>
</tr>
<tr>
<td>Mue2/10</td>
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<td>1.774</td>
<td>23500</td>
<td>1.6</td>
<td>+7.470</td>
</tr>
<tr>
<td>Mue2/11</td>
<td>0.0250</td>
<td>1.774</td>
<td>23500</td>
<td>1.6</td>
<td>+2.420</td>
</tr>
<tr>
<td>Mue2/14</td>
<td>0.0250</td>
<td>1.774</td>
<td>23500</td>
<td>1.6</td>
<td>+1.020</td>
</tr>
</tbody>
</table>
Applications of the new nonlinear slug test model

Measured vs. simulated responses at well Münstereifelbohrung B2:

Normalized Head $H(t)/H_0$

Data of test Mue2/1
Data of test Mue2/5
Data of test Mue2/7
Data of test Mue2/10
Data of test Mue2/11
Data of test Mue2/14
Simulations including all wellbore-internal head losses
Applications of the new nonlinear slug test model

Measured vs. simulated responses at well Münstereifelbohrung B2:

Time (sec.)

Normalized Head \( H(t)/H_0 \)

Data of test Mue2/1
Data of test Mue2/5
Data of test Mue2/7
Data of test Mue2/10
Data of test Mue2/11
Data of test Mue2/14
Simulations neglecting packer-related wellbore-internal head losses
Applications of the new nonlinear slug test model

Measured vs. simulated responses at well Münstereifelbohrung B2:

- Normalized Head $H(t)/H_0$
- Time (sec.)

Data of test Mue2/1
Data of test Mue2/5
Data of test Mue2/7
Data of test Mue2/10
Data of test Mue2/11
Data of test Mue2/14
Simulations neglecting rate-dependent skin losses
Applications of the new nonlinear slug test model

Measured vs. simulated responses at well Münstereifelbohrung B2:

- Data of test Mue2/1
- Data of test Mue2/5
- Data of test Mue2/7
- Data of test Mue2/10
- Data of test Mue2/11
- Data of test Mue2/14

Simulations neglecting rate-dependent skin losses and reducing $K_r$ from $4.2 \times 10^{-4}$ m/s to $2.0 \times 10^{-4}$ m/s
Inferences from Example No. 2:

- Nonlinear formation-controlled flow causes concavity and implies a shift of normalized head responses toward larger times similar to tubing-controlled flow.

- A differentiation of formation-controlled and tubing-controlled flow is possible but requires detailed knowledge of the geometric configurations of fittings (e.g. a packer) used inside the wellbore.

- Particularly, various-displacement well response testing can identify near-well nonlinear flow in fractured/karstified rock!

- Combine it with production logging or high-resolution borehole imaging techniques like FMI, UBI, AT to maximize structural and hydraulic information on investigated fractured systems.
Example No. 3 (a theoretical consideration):

Can fractional flow account for the observed head responses? Consider the fractional flow model of Barker (1988):

\[ S_s \frac{\partial s}{\partial t} = \frac{k}{r^{n-1}} \cdot \frac{\partial}{\partial r} \left( r^{n-1} \frac{\partial s}{\partial r} \right) \]
Applications of the new nonlinear slug test model

Simulations: Linear fractional flow model of Barker (1988) \( (n=1.0, k=2.0 \times 10^{-4} \text{ m/s}, S_s=1.0 \times 10^{-6} \text{ m}^{-1}, \text{ all nonlinear head losses neglected}) \)  
(Note: Simulated responses exactly collapse onto a unique convex curve)

Simulations performed as "pulsed" production/injection tests for a pumping/injection period of \( \Delta t=0.1 \text{ sec.} \) and corresponding rates:

\[
Q_{\text{pump}} = -\pi r_c c H_0 / \Delta t \text{ m}^3/\text{sec.}
\]
Applications of the new nonlinear slug test model

Data of test Mue2/1
Data of test Mue2/5
Data of test Mue2/7
Data of test Mue2/10
Data of test Mue2/11
Data of test Mue2/14

Simulations: Linear fractional flow model of Barker (1988) \( (n=1.5, k=1.0 \times 10^{-4} \text{ m/s}, S_s=1.0 \times 10^{-6} \text{ m}^{-1}, \text{all nonlinear head losses neglected}) \)
(Note: Simulated responses exactly collapse onto a unique convex curve)

Simulations performed as "pulsed" production/injection tests for a pumping/injection period of \( \Delta t=0.1 \text{ sec.} \) and corresponding rates:
\[ Q_{\text{pump}} = -\pi r_c r_d H_0 / \Delta t \text{ m}^3/\text{sec.} \]
Applications of the new nonlinear slug test model

Simulations: Linear fractional flow model of Barker (1988) \((n=2.5, k=3.0\times10^{-4} \text{ m/s}, S_s=1.0\times10^{-6} \text{ m}^{-1})\), all nonlinear head losses neglected) (Note: Simulated responses exactly collapse onto a unique convex curve)

Simulations performed as "pulsed" production/injection tests for a pumping/injection period of \(\Delta t=0.1 \text{ sec.}\) and corresponding rates:

\[ Q_{\text{pump}} = -\pi r_c r_c H_0 / \Delta t \text{ m}^3/\text{sec.} \]
Applications of the new nonlinear slug test model

Data of test Mue2/1
Data of test Mue2/5
Data of test Mue2/7
Data of test Mue2/10
Data of test Mue2/11
Data of test Mue2/14

Simulations: Linear fractional flow model of Barker (1988) (n=3.0, k=1.2×10^{-3} m/s, S_{s} = 1.0×10^{-6} m^{-1}, all nonlinear head losses neglected)
(Note: Simulated responses exactly collapse onto a unique linear curve)

Simulations performed as "pulsed" production/injection tests for a pumping/injection period of \( \Delta t = 0.1 \) sec. and corresponding rates:
\[ Q_{\text{pump}} = -\pi r_c c H_0 / \Delta t \text{ m}^3/\text{sec.} \]
Inferences from Example No. 3:

- Fractional flow does NOT provide concave normalized head responses in a Hvorslev-style format.
- Fractional flow does NOT yield shifted-in-time normalized head responses when using differing initial displacements $H_0$.
- Consequently, fractional flow is NOT the physical process causing the head responses observed at well Münstereifelbohrung B2.
Waterworks Beesen at Halle, Saxony-Anhalt

An example of characterizing a hydraulic barrier
Head responses are convex and reproducible for the weathered sandstone/mudstone unit; however, responses do not collapse onto a unique curve for different $H_0$. This may potentially be due to pseudo-plastic flow.
Summary

- Nonlinearity is evident from well response testing either by concave or by convex normalized head response curves, which are significantly shifted against one another when using varying displacements $H_0$. Fractional flow does not imply this head-dependent behavior.

- Nonlinear formation-controlled flow characteristics cannot be inferred from core analyses (poro-perm data) but may inexpensively be estimated by various displacement well response testing.

- Various displacement well response testing may especially be promising at:
  - hydraulic characterizations of fractured and karstified formations envisioned for drinking water supply and geothermal energy exploitation,
  - tightness characterization of fractured reservoir cap rocks and hydraulic barriers,
  - and potentially also for nuclear waste repository analyses.
--- Backup ---
Mechanical energy balance of the water inside the wellbore:

\[
- \left( H + z_0 + \left[ \frac{r_c^2}{r_p^2} - 1 \right] L_p + \left[ \frac{3}{8} B + D \right] \frac{r_c^2}{r_s^2} \right) \frac{d^2 H}{dt^2} - g \pi r_c^2 \left( B_2 + C \left( \pi r_c^2 \right)^{p-1} \right) \left| \frac{dH}{dt} \right|^{p-1} \frac{dH}{dt} \\
+ \frac{1}{2} \left[ \frac{r_c^2}{2r_s B} \right]^2 - 1 + \xi_{\text{loss}} - \left( f_p \left( \frac{L_p r_c^4}{2 r_p^5} \right) + f_s \left( \frac{D r_c^4}{2 r_s^5} \right) + f_c \left( \frac{z_0 - L_p + H}{2 r_c} \right) \right) \text{sign} \left( \frac{dH}{dt} \right) \left( \frac{dH}{dt} \right)^2 \\
- g \left( H - h \bigg|_{r=r_p} \right) = 0
\]
The basic aquifer response (away from the well) is assumed to be Darcian and cylindrically convergent toward the well:

\[
h|_{r=r_D} = -\frac{r_c^2}{2\pi^2 T} \int_0^t \frac{\partial^2 H}{\partial \tau^2} \int_0^\infty \frac{1-e^{\frac{4T(t-\tau)x^2}{Sr_D^2}}}{x^3 \left[ J_1^2(2x) + Y_1^2(2x) \right]} dxd\tau
\]

**Note:** The assumption of cylindrical flow convergence toward the tested well might not be correct for fractured rock settings in general. Any other (fractional) flow model could be used instead but a profound identification of the „true“ model governing formation flow might be difficult from slug testing alone.
Deviations from Darcian aquifer flow exist for:

- dominant electro-molecular forces (pre-linear laminar flow)
- dominant inertial effects due to flow path curvature, e.g. fracture flow channeling or flow in karstified rock (post-linear laminar flow)
- large flow rates in porous formations (Re > 100) resulting in turbulent flow (post-linear turbulent flow)
- significant fracture face roughness and sufficiently large flow rates (Re > 2400) resulting in turbulent fracture flow (post-linear turbulent flow)
- Non-Newtonian flow
Fracture flow channeling (from Kolditz, 2001):

Left: fracture roughness pattern ($b = \text{fracture aperture}$).
Right: simulated channelized velocity field.

by courtesy of Emerald Group Publishing Limited, 2012
Formation-related Nonlinearities:

A generalized rate-dependent skin effect is assumed to accommodate non-Darcian aquifer flow close to the well:

\[
\left. h \right|_{r=r_d} - H_{\text{Filter}} = \pi r_c^2 \left( B_2 + C (\pi r_c^2)^{p-1} \right) \frac{dH}{dt}^{p-1} \frac{dH}{dt}
\]

Note: The above rate-dependent skin formula projects additional head losses associated with the nonlinearity of formation flow in an empirical manner onto the wellbore face. Therefore, one cannot tell from an application of the above formula how far into the formation nonlinear flow would be significant.
Wellbore-associated Nonlinearities:

Colebrook’s formula is used to characterize turbulence inside the well:

\[
\frac{1}{\sqrt{f_{p,s,c}}} = -2.0 \cdot \log_{10} \left[ \frac{\varepsilon_{p,s,c}}{7.4 r_{p,s,c}} + \frac{2.51}{\text{Re}_{p,s,c} \sqrt{f_{p,s,c}}} \right]
\]
...and Borda Carnot-type head loss formulas to characterize minor head losses (shown here for the packer):

\[
\Delta p \left|_{\text{packer expansion}} \right. = -\frac{1}{2} \left(1 - \frac{r_p^2}{r_c^2}\right)^2 \left(\frac{r_c^2}{r_p^2}\right)^2 - \frac{dH}{dt} \left| \frac{dH}{dt} \right. = -\frac{1}{2} \xi_{\text{packer-expansion}} - \frac{dH}{dt} \left| \frac{dH}{dt} \right.
\]

\[
\Delta p \left|_{\text{packer contraction}} \right. = -\frac{1}{2} \cdot 0.42 \left(1 - \frac{r_p^2}{r_c^2}\right)^2 \left(\frac{r_c^2}{r_p^2}\right)^2 - \frac{dH}{dt} \left| \frac{dH}{dt} \right. = -\frac{1}{2} \xi_{\text{packer-contraction}} - \frac{dH}{dt} \left| \frac{dH}{dt} \right.
\]

\[
\xi_{\text{loss}} = -\left(\xi_{\text{packer-expansion}} + \xi_{\text{packer-contraction}}\right) \text{sign}\left(\frac{dH}{dt}\right)
\]
Data reproducibility: Repeated slug testing at well Münstereifelbohrung B2:
A Hvorslev-style variant of the nonlinear slug test model (Zenner, 2006):

\[- \left( H + z_0 + \left[ \frac{r_c^2}{r_p^2} - 1 \right] L_p + \left[ \frac{3}{8} B + D \right] \frac{r_c^2}{r_s^2} \right) \frac{d^2 H}{dt^2} - g \pi r_c^2 \left( B_2 + \frac{1}{F k_r} + C \left( \pi r_c^2 \right)^{p-1} \left| \frac{dH}{dt} \right|^{p-1} \right) \frac{dH}{dt} \]

\[+ \frac{1}{2} \left[ \left( \frac{r_c^2}{2 r_s B} \right)^2 - 1 + \xi_{loss} - \left( f_p \left( \frac{L_p r_c^4}{2 r_p^5} \right) + f_s \left( \frac{D r_c^4}{2 r_s^5} \right) + f_c \left( \frac{z_0 - L_p + H}{2 r_c} \right) \right) \text{sign} \left( \frac{dH}{dt} \right) \right] \left( \frac{dH}{dt} \right)^2 \]

\[- g H = 0 \]
A Hvorslev-style variant of the nonlinear slug test model (Zenner, 2006):

- $k_r = 3.0 \times 10^{-4}$ m/s
- $C = 13000$ sec.$^{p/m^{3p-1}}$
- $p = 1.5$
- Hvorslev's case no. 7
Validation against the model of Dougherty & Babu (1984) (fully penetrating case, with skin):

\[ \alpha = 2\left(\frac{r_d}{r_c}\right)^2 S = 0.0002 \]

- a) \( S_w = 0 \) \( B_2 = 0 \)
- b) \( S_w = 10 \) \( B_2 = 3978.873577 \)
- c) \( S_w = 120 \) \( B_2 = 47746.48293 \)

Symbols: Model of Dougherty & Babu (1984);
Solid lines: Nonlinear Slug Test Model
Backup: Model validation

Validation against the MLU-model of Hemker (1999) (one layer, no skin):

\[ \alpha = 2 \left( \frac{r_d}{r_c} \right)^2 S \]

a) \( \alpha = 0.018432 \)
b) \( \alpha = 7.3728 \times 10^{-6} \)
c) \( \alpha = 5.12 \times 10^{-14} \)

\[ L_{\text{eff}} = H_0 + z_0 + \left[ \frac{3}{8} B + D \right] \left( \frac{r_c}{r_s} \right)^2 \approx 0 \]
Validation against the MLU-model of Hemker (1999) (one layer, with skin):

\[ \alpha = 2 \left( \frac{r_D}{r_C} \right)^2 S = 0.001 \]

Solid lines: Nonlinear Slug Test Model

- a) \( S_w = 0 \) \( B_2 = 0 \)
- b) \( S_w = 25 \) \( B_2 = 1989.436788649 \)
- c) \( S_w = 100 \) \( B_2 = 7957.747154595 \)
References


References (continued)


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